Double unification, time compression, and space flatness for the extended particle

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Abstract: Three variational vector equations are derived for the extended particle-field object located on the light cone. Point sources are excluded from the pure field equations and all physical magnitudes are free from divergences. Accepting 3D intersections of 4D cone-charges, vector electrogravity explains all observed phenomena under common flat 3D and 1D intervals. External cone-charges contribute to pseudo-Riemannian metrics of proper four-spaces of charged objects, resulting in dilation-compression of their proper time rates. Photon-type gravitational radiation is not associated with metric modulation of flat, laboratory space. The Minkowski-Lorentz equation for free charges corresponds to the equivalence principle for the canonical four-space. The predicted criterion of double unification, particles with fields - gravity with electrodynamics, is confirmed in vector electrogravity.
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1. Introduction

Covariant equations for matter were originally derived for independent carriers of mass and charge [1]. But one elementary object N can carry both electric, $q_{_N}$, and gravitomechanical (mass), $m_{_N}$, charges. Gravity or acceleration can lead, for example, to a separation of opposite electric charges within an electroneutral medium with free electrons [2,3]. The induced electromagnetic fields under such separation depend essentially on the mass - charge ratio of carriers, while the mass of a carrier is not relevant in Maxwell's equations. The joint carrier for formally separated gravity and electromagnetism suggests that it is necessary to search for new variables for the classical Lagrangian of charged matter. The immediate task may be to derive at least one dynamic equation including the ratio of electric and gravitomechanical charges of material carriers.

The canonical four-momentum $P_{_N\mu} \equiv m_{_N} u_\mu + q_{_N} A_\mu^{\neq N} \equiv m_{_N}\mathcal{V}_\mu + m_{_N} U_\mu^{\neq N}$ seems to be one of the most appropriate notions for the description of a charged object N in its four-space with the proper metric tensor $g^{N}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + g^{\neq N}_{\mu\nu}(x)$ ($g^N_{\mu\nu}(x) \equiv g_{\mu\nu}$, for short; $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$), determined by all external objects K (*i.e.* K $\neq$ N, that is noted by $\neq N$). When the electromagnetic contribution is absent, $q_{_N} A_\mu^{\neq N} = 0$, the pure gravitomechanical four-momentum may be formally separated into mechanical and gravitational contributions, respectively,

$$m_{_N} u_\mu = m_{_N}\left\{\frac{1+(\sqrt{g_{oo}}-1)}{\sqrt{1-v_iv^i}}; -\frac{v_i+(\sqrt{g_{oo}}g_i)}{\sqrt{1-v_iv^i}}\right\} \equiv m_{_N}\mathcal{V}_\mu + m_{_N} B_\mu^{\neq N}, \qquad (1)$$

with the "curved" three-velocity $v_i = \gamma_{ij} v^j$ ($u_\mu = V_\mu \equiv g_{\mu\nu} dx^\nu / ds$ for $q_{_N} A_\mu^{\neq N}$ $= 0$, with $ds = (g_{\mu\nu} dx^\mu dx^\nu)^{1/2}$, $dx^\mu = dx^\mu_{_N}$, $v^i \equiv dx^i / g_{oo}^{1/2} (dx^o - g_i dx^i)$; $g_i =$ $-g_{oi}/g_{oo}$; $\gamma_{ij} = g_i g_j g_{oo} - g_{ij}$; $\mu, \nu \to 0, 1, 2, 3$; $i, j \to 1, 2, 3$; $c \equiv 1$).

By its natural involvement in various physical problems, the canonical four-momentum may be tried as a dynamical variable for the action of an elementary object N. But the classical theory of fields and particles, for example [4-8], does not employ the canonical four-momentum as a dynamic variable. The known approaches, for example [7], to combine mechanical and electric charges under a joint geodesic motion were associated with complicated modifications of space geometry and discussions about the structure of charges in general relativity. For well known reasons the classical theories of fields and particles, including the non-dualistic approaches [9-12], look incomplete and do not overcome some internal difficulties.

We examine a non-dualistic way once again by trying to exclude point sources (associated with observations) from the field equations in agreement with Einstein's intention. The particle integration into the very structure of the field was assumed in the last Einstein constructions, for example, "We could regard matter as being made up of regions of space in which the field is extremely intense... There would be no room in this new physics for both field and matter, for the field would be the only reality" (translation [11]). This program is not accomplished yet in a classical approach and it may be considered as a motivation for our efforts.

In order to reveal the new opportunities of the classical theory we replace the point charge by the elementary charged continuum emanating from a point source in parallel with the Coulomb and the Newton fields. This elementary field continuum with homogeneous charge densities at the proper light cone-horn points may be called (conventionally) a cone-particle. At first glance this alternative approach would seem unreasonable in any practical treatment because every infinite charged continuum of matter would have infinite energy. But it will be shown below that the emanating cone-particle and the paired emanating cone-field together form a unified material complex, called the elementary particle-field object, with zero components of the energy-tensor density ($T^{\mu\nu}_{_N} = 0$ for superfluid, potential motion). Einstein's concept of cone-charges integrated into "the very field structure" becomes free from infinite self-energies. This concept can propose a clear mechanism for a particle's action-at-a-distance [12], when extended charges, not sources, interact locally at all points of the common space-charge-mass 3D continuum.

Again, we start from the assumption that every elementary charge or particle may be considered in terms of an infinite material continuum (emanating with a zero four-interval from a moving point source). Each of the two mirror cones with the joint vertex in four-space contains its own particle matter to counterbalance its own elementary field. This assists us in removing from the theory the unreasonable advanced field solutions of classical electrodynamics, where only one point particle-source (rather than two mirror particle-sources) in a joint vertex was wrongly associated with both Minkowski's cones.

The retarded functions appear in the theory with locally bound particle-field matter (at all cone points x) only with respect to its source at cone's vertex ξ. The emitting material cone object (excluding its vertex) may be treated as one multifractional field in non-dualistic terminology, but we shall refer traditionally to the elementary particle and to the elementary field fractions

in order to trace their contributions to the pure field equations (derived below). But now cone-particles are not complete elementary objects as they cannot move independently from their cone fields, located on the same points of the light cone.

We shall introduce a unified particle-field action for one elementary cone object N, for which we employ the proper vector variables, $P_{_N\mu}$ and $a_{_N\mu}$, associated with external, $m_{_K}a_{_K\mu}$, $e_{_K}a_{_K\mu}$ and proper, $m_{_N}a_{_N\mu}$, $e_{_N}a_{_N\mu}$, fields, respectively. The Euler-Lagrange equations for the extended cone object will involve only finite physical magnitudes, and these four-vector equations will correspond to the known demands for observed motion of charges. Electrodynamics and gravitation appear as a unified non-dualistic field theory, where both the Maxwell-type equations and the vector replacement of the Einstein-type equation follow directly from one variational equation, $P_{_N\mu}T^{\mu\nu}_{_N} = 0$.

The inseparably bound particle and field fractions of elementary matter will assist us in overcoming the classical problem of charged particle self-acceleration after replacing the Minkowski equation (with the Lorentz force) with its generalization, $m_{_N}DP_{_N\nu}/ds_{_N} = P^{\mu}_{_N}(\nabla_\mu P_{_N\nu} - \nabla_\nu P_{_N\mu}) = 0$, for geodesic motion in the proper canonical four-space $x_{_N}$, where $P_{_N\mu} = m_{_N}g^{_N}_{\mu\nu}dx^{\nu}_{_N}/ds_{_N}$.

After deriving the field equations in terms of the proper field densities for every extended cone object we shall verify the symmetrical involvement of external electric charge and mass densities (associated with a joint forming-up field $a_{_K\mu}$) into the proper canonical four-momentum density $P_{_N\mu}$. This will reveal new (electromagnetic) references for the metric tensor $g^{_N}_{\mu\nu}$ of the proper pseudo-Riemannian four-space.

We shall employ the accepted tetrad formalism in order to demonstrate the hidden metric symmetry, $\gamma^{_K}_{ij} = \delta_{ij}$, in the three-interval $dl^2 = \gamma^{_K}_{ij}dx^i_{_K}dx^j_{_K}$ of any material object K, despite the fact that every component of the proper pseudo-Riemannian metric tensor, $g^{_K}_{\mu\nu} \neq \eta_{\mu\nu}$, represents gravity in full agreement with the Einstein covariant scheme. This finding will provide the opportunity to introduce the common (for all objects) space+time manifold $\{\mathbf{x}; dt\}$, with the universal, flat time interval, $dt^2 = \gamma^{_K}_{oo}dx^o_{_K}dx^o_{_K}$ and $\gamma^{_K}_{oo} = \delta_{oo}$ for all cone-particles K crossing the selected point $\mathbf{x}$ of the united space-charge-mass continuum. The six bounds $\gamma^{_K}_{ij} = g_{oi}g_{oj}g^{-1}_{oo} - g_{ij} = \delta_{ij}$ for ten metric components $g^{_K}_{\mu\nu}$ will lead finally to four, not ten, independent variational equations for vector gravitation.

It will be remarkable to derive that the covariant applications of Euclidean 3D subspace within curved four-space are consistent with the planet perihelion precession, gravitational light bending and time dilation. The flat three-space acknowledges selection of the proper mechanical contribution $m_{_N}\mathcal{V}_\mu \equiv m_{_N}(1 - \delta_{ij}v^iv^j)^{-1/2}\{1; -v_i\}$ and the gravitational contribution $m_{_N}B^{\neq N}_\mu \equiv m_{_N}(1 - \delta_{ij}v^iv^j)^{-1/2}\{\sqrt{g_{oo}} - 1; -\sqrt{g_{oo}}g_i\}$ in the gravitomechanical momentum (1).

We shall study the hitherto unexplained relativistic experiments with rotating superconductors [13, 14] in order to demonstrate the applications of the introduced extended charges for solid state physics. One could select the other experimental indications against the point model of elementary electric charge and mass, including the celebrated Aharonov - Bohm phenomenon [15].

Gravitational interactions in vector electrogravity (VEG) are associated with linear synthesis of external vector fields, $B^{\neq N}_\mu = -G\sum^{K\neq N}_{K} m_{_K}a_{_K\mu}$, where every forming-up four-vector potential $a_{_K\mu}$ is determined by its proper field equation. Electrodynamic interactions with external fields are also associated with the

same forming-up potentials, with $A_\mu^{\neq N} = \sum_K^{K\neq N} e_K a_{K\mu}$. The mass is a source of gravity in the present scheme, rather than the energy tensor density.

New opportunities of electrogravity with the extended masses and flat 3D subspace allow this theory to incorporate electrical charges into the standard covariant scheme with the proper canonical pseudo-Riemannian four-space. External mass and electric charge densities cannot change Euclidean geometry of the proper 3D subspaces, but they affect the proper time of every charged object. The predicted electromagnetic time dilation-compression is available for simple laboratory tests. Both electromagnetic and gravitational waves in the present scheme are vector photons, contrary to the tensor gravitational wave of general relativity, and there are no metric modulations of flat, laboratory three-space.

2. Action of the extended particle-field object

It is common knowledge that the covariant electrodynamic equations with a current density and with the Lorentz force may be obtained from the variational principle in four-space. Both relativistic methods, developed by M. Born [16] and H. Weyl [17], state that it is possible to fix electromagnetic fields under path variations for charges, as well as to fix four-coordinates of free charges under field variations. But such assumptions cannot be valid in general. Sometimes a coordinate displacement of charges is the only reason for the creation of macroscopic electromagnetic fields within an electroneutral system (a rotating conductor, for example).

The particular purpose of this section is to propose the universal dynamic variables in order to eliminate the preliminary assumptions one uses when varying the action of charged matter. For this purpose we consider for a moment the pure particle action-at-a-distance $\mathcal{S}_N^p$ with one point particle-source [12],

$$\mathcal{S}_N^p = -\int [\hat{m}_N(\xi_N[p_N]) u_\mu(\xi_N[p_N]) + \hat{q}_N(\xi_N[p_N]) A_\mu^{\neq N}(\xi_N[p_N])] \frac{d\xi_N^\mu[p_N]}{dp_N} dp_N$$
$$\equiv -\int dp_N \left\{ \hat{m}_N u_\mu + \hat{q}_N \sum_K^{K\neq N} \int dp_K \hat{q}_K(\xi_N) \eta_K(\xi_N, \xi_K)_{\xi_N\neq\xi_K} \frac{d\xi_{K\mu}}{dp_K} \right\} \frac{d\xi_N^\mu}{dp_N}, \quad (2)$$

where the selected source N and all other sources K = 1, 2, ..., N-1, N+1, ... are associated, respectively, with gravitomechanical charge-sources $\hat{m}_N(\xi_N[p_N])$ and $\hat{m}_K(\xi_K[p_K])$, electric charge-sources $\hat{q}_N(\xi_N[p_N])$ and $\hat{q}_K(\xi_K[p_K])$, coordinates $\xi_N^\mu[p_N]$ and $\xi_K^\mu[p_K]$, which may be regarded as functions of parameters p_N and p_K on their classical paths $\xi_N \equiv \xi_N[p_N]$ and $\xi_K \equiv \xi_K[p_K]$.

The local interaction of a point source N at the point ξ_N with the field cone-charge density $q_K(\xi_N)^{s_K=0}_{\xi_N\neq\xi_K} = q_K = const$ of an external cone object K is determined by the basic operator $\eta_K(\xi_N[p_N], \xi_K[p_K])_{\xi_N\neq\xi_K}$. Any different point sources cannot have all four common coordinates and this is indicated in the symbolic form $\xi_N \neq \xi_K$. The basic operator $\eta_K(\xi_N[p_N], \xi_K[p_K])_{\xi_N\neq\xi_K}$ may be specified on an infinite proper four-space x_K, which intersects, in particular, the point $\xi_N[p_N]$. We accept continuous coordinates $x_K^\nu = \{x_K^o; x_K^i\}$ for every proper four-space with the proper metric tensor $g_{\mu\nu}^K(x_K)$. Intersections of the curved proper four-spaces, due to their joint 3D subspaces, acknowledge an introduction of the common space+time, $\{dt; \mathbf{x}\}$, for an ensemble of material

objects after an appropriate application of the common time interval dt (defined below) and the common three-space (which ought to keep universal geometry for all intersecting subspaces). Below there will appear two opposite parametric time intervals, $dt^{K}_{1,2}(\mathbf{x}) = \pm|dx^{o}_{K}|$, for mirror three-dimensional motion of any selected object and antiobject N contrary to the accepted Minkowski approach with $dt \equiv dx^{o}$ for all cases. Moreover, dt will not be a component of any proper vector dx^{ν}_{K} in spite of $dt^2 = (dx^{o}_{K})^2$. This parametric interval dt will be introduced for a considered point in flat three-space, where cone-particles (located on different pseudo-Riemannian four-spaces x_K - light cones, for shot), can intersect.

Unlike the formal interaction-at-a-distance between point sources, a real interaction of infinite cone objects takes place locally at the intersection of the emitted cones at joint material points. The basic operator $\eta_K(x_K, \xi_K[p_K])_{x_K \neq \xi_K}$ will be responsible for the local interaction of the selected object N with the external object K at joint material points $x_N = x_K$ under the zero four-intervals $s_N(x_N, \xi_N) = 0$ and $s_K(x_K, \xi_K) = 0$, with $x_N \neq \xi_N$ and $x_K \neq \xi_K$.

By making use of the equality (2) and of the proper four-space notion $x_N \equiv x$, one can introduce for every elementary object N a covariant four-potential of the external electromagnetic field $A^{\neq N}_{\mu}(x) \equiv A^{\neq N}_{\mu}(x, \xi_1, \xi_2, ..., \xi_{N-1}, \xi_{N+1}, ...) \equiv \sum^{K \neq N}_{K} \mathcal{A}_{K\mu}(x)^{s_K[\tau_K]=0}_{x \neq \xi_K[\tau_K]}$, where τ_K is a "material" value of the path parameter p_K of an elementary cone object K when it crosses the considered point x. The four-potential $\mathcal{A}_{K\mu}(x)^{s_K[\tau_K]=0}_{x \neq \xi_K[\tau_K]}$ of the elementary electromagnetic field of any charged cone object K at its material point x_K, with $x_K = x$, is related to a four-space position of a source K at the point ξ_K by the zero four-interval, $s_K(x, \xi_K[\tau_K]) = 0$, with $x \neq \xi_K$, that determines the "material" parameter τ_K.

Note, that different points x correspond to different "material" values of the path parameters of the same object, *i.e.* $\tau_K \equiv \tau_K(x)$. One should use the zero-intervals in determining the elementary electromagnetic four-potential $\mathcal{A}_{K\mu}(x)^{s_K[\tau_K]=0}_{x \neq \xi_K[\tau_K]} \equiv \int dp_K q_K(x) \eta_K(x, \xi_K[p_K])_{x \neq \xi_K[p_K]} \{d\xi_{K\mu}[p_K]/dp_K\}$ or the four--potential $a_{K\mu}(x)^{s_K[\tau_K]=0}_{x \neq \xi_K[\tau_K]} \equiv \int dp_K \eta_K(x, \xi_K[p_K])_{x \neq \xi_K[p_K]}\ \{d\xi_{K\mu}[p_K]/dp_K\}$ for the basic (forming-up) uncharged field of every elementary object K, which contributes to the total material field at the considered point x. Only retarded zero-interval relations with sources will appear for emitted cone continua after an appropriate use of the two space+times $\{dt_{1,2}, x^i\}$ (one for matter, the other for antimatter), rather than the accepted four-space manifold $\{x^o, x^i\}$.

The proper field (or potential) $a_{N\mu}(x)^{s_N[\tau_N]=o}_{x \neq \xi_N[\tau_N]}$ at the considered point x was emitted by the source at one of its path points, $\xi^{\mu}_{K}[p_N]$, which cannot be defined without referring to the equation of motion (derived after variations). This four-vector field takes four degrees of freedom and may be a dynamic variable for a cone object N at all material points $x = x_N$ of its curved light cone-horn.

A certain source position $\xi_N \equiv \xi$, for short, may be conjugated (through a zero four-interval) with the material field points by a defining relation $\hat{Q}_N(\xi) \equiv \int d^4x Q_N(x)\ \hat{\delta}^4_N(x, \xi)_{x \neq \xi}$. The accepted pseudo-geometry (zero-interval matter) defines the structure of the operator $\hat{\delta}^4_N(x, \xi[p])_{x \neq \xi[p]} \equiv \{\hat{\delta}^3_N(\mathbf{x}, \boldsymbol{\xi}[p])\ \delta_N(x^0 - \zeta^0[p])\}_{x \neq \xi}$, where the effective coordinate $\zeta^0[p]$ should be determined by the "time" coordinate $\xi^0[p]$ and by the "time" delay (for flat four-space $\zeta^0[p] = \xi^0[p] \pm|\mathbf{x} - \boldsymbol{\xi}[p]|$). By noting $x \neq \xi$ we emphasize below that the continuous function-densities represent emitted cone matter at any considered point x of four-space

but not a source at the vertex ξ, which is a nonmaterial peculiarity within this elementary material continuum. A similar statement is true for material cone-particle for points $\mathbf{x}$ and source points $\boldsymbol{\xi}$ when it is noted that $\mathbf{x} \neq \boldsymbol{\xi}$ for three-space.

One ought to exclude the source point ξ (and $\boldsymbol{\xi}$) from an elementary cone-particle in order to avoid a twofold account of the conjugated notions (the nonmaterial source and the associated cone-particle) under description of one elementary object. To operate with two different kinds of coordinates for point sources of matter and for matter itself (*i.e.* infinite particle-field cones excluding the vertexes), we have to distinguish the conjugated characteristics. For example, a function $\hat{P}_{{}_N\mu}(\xi_{{}_N}[p_{{}_N}])$ represents formally a canonical four-momentum of a point source N. A function-density $P_{{}_N\mu}(x)^{s=o}_{x\neq\xi}$ is a canonical four-momentum density for a real particle-cone N at its material points $x \equiv x_{{}_N}$, when $x \neq \xi[\tau]$ and $s(x, \xi[\tau]) = 0$ (and for a virtual particle N at all points $x \neq \xi[p]$ if $s(x, \xi[p]) \neq 0$).

The gravitomechanical, $m_{{}_N}(x)^{s=0}_{x\neq\xi}$, or the electric, $q_{{}_N}(x)^{s=0}_{x\neq\xi}$, elementary charge density of the cone-particle is conjugated to the point particle-source mass, $\hat{m}_{{}_N}(\xi)$, or electric charge, $\hat{q}_{{}_N}(\xi)$, respectively, $\int d^4x m_{{}_N}(x)\hat{\delta}^4_{{}_N}(x,\xi)_{x\neq\xi} \equiv \hat{m}_{{}_N}(\xi)$ or $\int d^4x q_{{}_N}(x)\hat{\delta}^4_{{}_N}(x,\xi)_{x\neq\xi} \equiv \hat{q}_{{}_N}(\xi)$. The scalar cone-charge densities $m_{{}_N}(x) \equiv m_{{}_N}$ and $q_{{}_N}(x) \equiv q_{{}_N}$ are homogeneous functions of the four-space coordinates, $\partial_\mu m_{{}_N}(x) = 0$ and $\partial_\mu q_{{}_N}(x) = 0$, as well as the charge-source functions $\hat{m}_{{}_N}(\xi[p])$ and $\hat{q}_{{}_N}(\xi[p])$ are independent of the path coordinates $\xi[p]$. This provides linear relations for the elementary electromagnetic, $\mathcal{A}_{{}_N\mu}(x)^{s=0}_{x\neq\xi} \equiv q_{{}_N} a_{{}_N\mu}(x)^{s=0}_{x\neq\xi}$, and the elementary gravitomechanical, $\mathcal{B}_{{}_N\mu}(x)^{s=0}_{x\neq\xi} \equiv m_{{}_N} a_{{}_N\mu}(x)^{s=0}_{x\neq\xi}$, material fields because the densities $q_{{}_N}$ and $m_{{}_N}$ are universal constants.

The Green's structure of the basic operator $\eta_{{}_N}(x,\xi[p])_{x\neq\xi[p]}$ will be described below in the Appendix 1. What is important to underline right now is that the proper canonical four-momentum density of the cone-particle, $P_{{}_N\mu}(x)^{s=o}_{x\neq\xi} \equiv \{m_{{}_N}(x)u_\mu(x) + q_{{}_N}(x)A^{\neq N}_\mu(x)\}^{s=o}_{x\neq\xi}$, is independent of the forming-up uncharged cone-field at every considered point x of the elementary particle-field object N. This means that $P_{{}_N\mu}(x)^{s=o}_{x\neq\xi}$ and $a_{{}_N\mu}(x)^{s=o}_{x\neq\xi}$ might be independent dynamic variables for the description of the same elementary cone object N.

The particle-source action (2) is independent of the emitted field fraction of matter and can be associated only with the pure particle fraction of the cone object. But the particle is always accompanied by its own field, *i.e.* every particle is only a fraction of an infinite particle-field object. Then, a complete action $\mathcal{S}^{pf}_{{}_N}$ of this self-contained object should be contributed to by both the particle and the field elementary fractions.

Before adding the paired elementary cone-field to the action (2), we introduce a canonical tensor density $W_{{}_N\mu\nu}(x)^{s=o}_{x\neq\xi} \equiv \{\nabla_\mu P_{{}_N\nu}(x) - \nabla_\nu P_{{}_N\mu}(x)\}^{s=o}_{x\neq\xi} = \{\partial_\mu P_{{}_N\nu}(x) - \partial_\nu P_{{}_N\mu}(x)\}^{s=o}_{x\neq\xi}$ of the elementary cone-particle N. The covariant derivatives, ∇_μ, may be replaced in the vorticity $W_{{}_N\mu\nu}(x)^{s=o}_{x\neq\xi}$ by the partial ones, ∂_μ, due to the symmetry of the Christoffel coefficients in the proper four-space $x^\mu_{{}_N}$ with respect to the proper four-vectors $P_{{}_N\mu}$ and $a_{{}_N\mu}$ (not with respect to $P_{{}_K\mu}$ and $a_{{}_K\mu}$, which are four-vectors in other curved four-spaces). The cone-field contribution to the complete action $\mathcal{S}^{pf}_{{}_N}$ of the elementary particle-field object N can be introduced in terms of the scalar Lagrangian density within a

four-dimensional volume,

$$\mathcal{S}^{pf}_{N} = -\int dp \hat{P}_{N\mu}(\xi[p]) \frac{d\xi^\mu[p]}{dp} - \int \frac{\sqrt{-g_N} d^4x}{16\pi} \{ g^{\mu\rho}_N g^{\nu\lambda}_N W_{N\mu\nu}(x) f_{N\rho\lambda}(x) \}^{s[\tau]=o}_{x\neq\xi[\tau]}, \tag{3}$$

where both the new tensor density $f_{N\mu\nu}(x)^{s[\tau]=o}_{x\neq\xi[\tau]} \equiv \{\nabla_\mu a_{N\nu}(x) - \nabla_\nu a_{N\mu}(x)\}^{s[\tau]=o}_{x\neq\xi[\tau]}$ $= \{\partial_\mu a_{N\nu}(x) - \partial_\nu a_{N\mu}(x)\}^{s[\tau]=o}_{x\neq\xi[\tau]}$ for the elementary cone-field and the canonical tensor density $W_{N\mu\nu}(x)^{s[\tau]=o}_{x\neq\xi[\tau]}$ for the elementary cone-particle are accompanied by the material restrictions $x \neq \xi[\tau]$ and $s(x,\xi[\tau]) = 0$ for an elementary cone object N, when it crosses every point x used in (3) for the four-dimensional integration, *i.e.* when $x \equiv x_N$. Hereinafter $g^{\mu\nu}_N(x) \equiv \eta^{\mu\nu} + g^{\mu\nu}_{\neq N}(x)$, with $g^{\mu\nu}_N g_{N\mu\lambda} = \delta^\nu_\lambda$.

So far, the first item in the action (3) corresponds formally to a point "charged" source, rather than to a charged cone-particle. One may interchangeably rewrite the complete action via the operators for a virtual object N, which gains its real cone state (crossing the considered point x) only after integration in (3) over the path parameter p,

$$\mathcal{S}^{pf}_{N} = -\int dp \int \sqrt{-g_N} d^4x \{ P_{N\mu}(x) \frac{dx^\mu}{dp} \frac{\hat{\delta}^4_N(x,\xi[p])_{x\neq\xi[p]}}{\sqrt{-g(x)}}$$

$$+ \frac{g^{\mu\rho}_N g^{\nu\lambda}_N W_{N\mu\nu}(x)}{16\pi} \left[\frac{d\xi_\lambda[p]}{dp} \frac{\partial\eta_N(x,\xi[p])}{\partial x^\rho} - \frac{d\xi_\rho[p]}{dp} \frac{\partial\eta_N(x,\xi[p])}{\partial x^\lambda} \right]_{x\neq\xi[p]} \}. \tag{3a}$$

Different points x in four-space can be occupied by the same material particle - field cone, $s(x,\xi[\tau]) = 0$, under different source locations and under different "material" values, $\tau \equiv \tau(x)$, of the path parameter p. The action (3a) may be formally simplified in terms of real cone matter after integration (3a) over p,

$$\mathcal{S}^{pf}_{N} = -\int \sqrt{-g_N} d^4x \left\{ P_{N\mu}(x) i^\mu_N(x) + \frac{g^{\mu\rho}_N g^{\nu\lambda}_N W_{N\mu\nu}(x) f_{N\rho\lambda}(x)}{16\pi} \right\}^{s[\tau]=o}_{x\neq\xi[\tau]}, \tag{3b}$$

where we introduced the four-flow density,

$$i^\mu_N(x)^{s[\tau]=o}_{x\neq\xi[\tau]} \equiv \int \frac{dx^\mu}{dp} \frac{\hat{\delta}^4_N(x,\xi[p])_{x\neq\xi[p]}}{\sqrt{-g_N(x)}} dp, \tag{4}$$

of the elementary cone-particle N at any of its material points $x \equiv x_N$, selected for consideration. But one should keep in mind for formal action (3b) that all variations of the action have to be done with respect to both virtual and material states of the object N, *i. e.* variations have to be considered before the integration over the parameter p. After the integration over p the independent variables, like $P_{N\mu}$ and x^μ, may be bound by additional restrictions for material states.

Accepting approach with action for one selected object N, we ought to distinguish contributions from the proper, $a_{N\mu}$, and external, $a_{K\mu}$, fields. The proper four-momentum $P_{N\mu}(x)$ and the proper metric tensor $g_{\mu\nu} = g^N_{\mu\nu}(x)$ are independent from $a_{N\mu}$, but they both depend on the same system of external objects K with their proper fields $a_{K\mu}(x)$. That is the reason that the proper

canonical momentum and the proper metric tensor are related variables (their links with the same external field $U_\mu^{\neq N} = \sum_K^{K\neq N} c_K a_{K\mu}$ will be revealed at the final steps of the below developed scheme for canonical four-space, where $P_{N\mu} P_{N\nu} g^N_{\mu\nu} = m^2_N$). One may assume for a moment that the external field $U_\mu^{\neq N}$ is to be a final variable for the object N, rather than $P_N\mu$ or $g^{-N}_{\mu\nu}$. But the sum $U_\mu^{\neq N}$, as well as each summand $a_{K\mu}$, is not a four-vector in the proper space x^μ_N ($a_{K\mu} \neq g_{N\mu\nu} a^\nu_K$ when K $\neq$, for example). The proper four-vector $P_{N\mu}$ can perfectly represent the external Newton-Coulomb fields in the action (3)-(3b) because we shall find that $\delta P_{N\mu} = \delta U_\mu^{\neq N}$ and there will be no tensor fields in the present approach to gravitation. Finally only proper coordinates, x^μ_N, and two proper four-vectors, $a_{N\mu}(x)$ and $P_{N\mu}$, may be selected as independent variables in the developed theory with the vector variational equations.

One may propose to study an ensemble of elementary objects and to consider a summary action $\sum_N \mathcal{S}^{pf}_N$. But there is no universal four-space for all elementary objects and, contrary to classical electrodynamics, there is no way to introduce a collective field variable in the present scheme. It is more accurate to operate with a system of the Euler - Lagrange equations, derived below for every elementary object in its external fields, rather than to speculate about a joint action and possible collective variables for an ensemble of different objects.

3. Universal time interval

The variational procedure in (3a) with respect to the canonical four-momentum density, both real and virtual variations $\delta P_{N\mu}(x)$, with $\delta g^{\mu\nu}_N = 0$, can lead to a Maxwell-type equation for a total four-flow density, $I^\nu_N(x)^{s[\tau]=0}_{x\neq\xi[\tau]}$, of the elementary cone object,

$$I^\nu_N(x)^{s[\tau]=0}_{x\neq\xi[\tau]} \equiv \left\{ i^\nu_N(x) - \frac{\nabla_\mu [f^{\mu\nu}_N(x) - f^{\nu\mu}_N(x)]}{8\pi} \right\}^{s[\tau]=0}_{x\neq\xi[\tau]} = 0, \qquad (5)$$

where $f^{\mu\nu}_N(x)^{s=0}_{x\neq\xi} \equiv g^{\mu\rho}_N(x) g^{\nu\lambda}_N(x)\ f_{N\rho\lambda}(x)^{s=0}_{x\neq\xi} = -f^{\nu\mu}_N(x)^{s=0}_{x\neq\xi}$. Note that not all components of the skew-symmetric tensors are independent under variation [11]: the relations $\delta W_{N\mu\nu}(x) = -\delta W_{N\nu\mu}(x)$ must be taken into account.

So far we do not know the final relations of $P_{N\mu}$ and $g^{\mu\nu}_N$ with external fields, and we temporary neglect the relation between δP^N_μ and $\delta g^{\mu\nu}_N$. For this reason the Maxwell-type equation (5) is incomplete in our approach (this simplified equation will be accepted only for potential states, when $W_{N\mu\nu}(x)^{s=0}_{x\neq\xi} = 0$).

The arbitrary variations are not necessarily compatible [18] with any restricting conditions for the path parameter p, for example $s[p] \neq 0$ and $P_{N\mu} P^\mu_N \neq m^2_N$ under virtual variations in (3a). But after variation of the action, one may specify the appropriate path parameter in the derived equations of motion due to some additional restrictions for real matter (or for real antimatter). In equation (5) we operate with the family of material points x which correspond to the real cone object N. A selection of any one point x provides an appropriate selection of the path parameter $p = \tau_{1,2}$ due to the material restriction $s[\tau_{1,2}] = 0$ with two possible solutions τ_1 and τ_2 for the mirror cones in the metric four-space.

Even though the covariant equations are four-dimensional in the proper four-space, dynamics of matter depends on the development parameter, and there must be a three-dimensional picture as seen by an observer. This motivates

us to introduce a new parametric interval dt in order to describe the evolution of matter (or antimatter) in three-space $\mathbf{x}$. One can therefore perform the integration over p in the definition (4) and to introduce the material four-flow densities of two mirror cone-particles via the appropriate time differentials dt_1 and dt_2,

$$i^{\nu}_{N}(x)^{s[\tau_{1,2}]=0}_{x\neq\xi[\tau_{1,2}]} \equiv \int \frac{dx^\nu}{dp}\frac{\{\hat{\delta}^3_N(\mathbf{x},\boldsymbol{\xi}[p])\delta_N(x^0-\zeta^0[p])\}_{x\neq\xi[p]}}{\sqrt{-g}}dp$$

$$= \int \frac{dx^\nu}{\sqrt{-g}dp}\frac{\{\hat{\delta}^3_N(\mathbf{x},\boldsymbol{\xi}[p])\delta_N(p-\tau_{1,2})\}_{x\neq\xi[p]}}{\left|\frac{\partial\zeta^0[p]}{\partial p}\right|_{x\neq\xi[p]}}dp =$$

$$\left\{\frac{\hat{\delta}^3_{N1,2}(\mathbf{x},\boldsymbol{\xi}[\tau_{1,2}])dx^\nu}{\sqrt{\gamma}\sqrt{g_{oo}}d\tau_{1,2}\left|\frac{\partial\zeta^0[\tau_{1,2}]}{\partial\tau_{1,2}}\right|}\right\}^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]} \equiv \left\{\frac{\hat{\delta}^3_{N1,2}(\mathbf{x},\boldsymbol{\xi})}{\sqrt{\gamma}}\frac{dx^\nu}{\sqrt{g_{oo}}dt_{1,2}}\right\}^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}, \quad (6)$$

where $\gamma \equiv ||\gamma_{ij}|| = -g/g_{oo}$. The operators $\hat{\delta}^3_{N1,2}(\mathbf{x},\boldsymbol{\xi}[\tau_{1,2}])^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}$ conjugate function-paths of the point sources in three-space ($\boldsymbol{\xi}[\tau_{1,2}] \equiv \boldsymbol{\xi}_{N1,2}[\tau_{1,2}]$ for the mirror particle-sources N_1 and N_2) to function-densities of the mirror cone-particles N_1 and N_2 projected onto three-space $\mathbf{x}$.

Every considered point $\mathbf{x}$ with coordinates x^i in the 3D subspace of the proper four-space $x^\nu_N \equiv x^\nu$ can be related to two source points $\xi[\tau_{1,2}]$, $\xi[\tau_1] \neq \xi[\tau_2]$, by zero-interval conditions, $s(x,\xi[\tau_1]) = 0$ and $s(x,\xi[\tau_2]) = 0$. Both these conditions provide $dx^o = d\zeta^o[\tau_{1,2}]$, where $d\zeta^o[\tau_{1,2}] = d\tau_{1,2}\partial\zeta^o[\tau_{1,2}]/\partial\tau_{1,2}$ for the mirror cones with matter or antimatter. By making use of this fact we introduced in (6) the opposite parametric differentials at every point $\mathbf{x}$,

$$dt_{1,2}(\mathbf{x}) \equiv \left\{\frac{d\tau_{1,2}}{|d\tau_{1,2}|}\left|d\tau_{1,2}\frac{\partial\zeta^0[\tau_{1,2}]}{\partial\tau_{1,2}}\right|\right\}^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]} = \{sign(d\tau_{1,2})|dx^o|\}^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}, \quad (7)$$

which may be called the direct and the inverted time intervals, $dt_1(\mathbf{x})$ and $dt_2(\mathbf{x})$, respectively. The parametric intervals $dt_{1,2}(\mathbf{x},\tau_{1,2})$ are introduced for the cone-particles (cone-charges) crossing a particular point in three-space $\mathbf{x}$. It is important for the anticipated description of an ensemble that both these intervals are finally independent from the proper parameters of different objects and the universal time interval may be employed for all cone-particles with rest masses.

The direct, with τ_1, (the inverted, with τ_2) four-flow density (6) of a cone-particle and the direct (the inverted) time interval (7) are associated with the direct (the inverted) elementary cone-field in (5) (Appendix 1). By applying three-space and the time parameter from (7), one finds a coincidence of a dynamic three-dimensional picture for direct particle-field objects (matter) and for inverted ones (antimatter) under appropriate applications of the direct and the inverted time intervals, $dt_1 = -dt_2 = |dx^o|$, for example. The appearance of two opposite time intervals (7) with parametrically oriented directions provides an opportunity to introduce two parallel space+time manifolds, $\{dt_1, x^i\}$ and $\{dt_2, x^i\}$, on the basis of one four-space metric system $\{x^o, x^i\}$. This allows one to trace the bound charge-time contribution into Charge-Parity-Time symmetry and to explain the PT symmetry violation.

The mirror elementary cone-particles N_1 and N_2 occupy the direct and the inverted cones with matter and antimatter, respectively, within one metric four-space. But a particle (or antiparticle) fraction from one cone is not bound with an antifield (or field) fraction from the mirror cone. By trying to relate one point charged particle in the joint vertex of two pure field cones to both these fields, the Minkowski theory resulted in the unreasonably advanced solutions for emitted field matter.

There are neither retarded nor advanced relations of the cone-particle with the paired cone-field in the concept of extended cone-charges. The cone-field and cone-particle elementary densities are locally bound (without any delay) at every material point in four-space. By choosing appropriately the space+time manifolds for matter or for antimatter, one obtains only retarded emission from point sources. Below we omit the "1" or "2" subscript in $d\tau$ or dt by dealing, for simplicity, only with matter in the direct space+time manifold $\{dt, x^i\}$, $dt = +|dx^o|$, for example.

Again, the choice of the time parameter t for matter or antimatter is irrelevant. The important point is that the cone-particle four-flow density may be divided in (5) into the direct and the inverted components, as well as the elementary cone-field at the left hand side of the equation (5). Note, that the Dirac operator $\delta^4_{N}(x-\xi) \equiv \delta^3_{N}(\mathbf{x}-\boldsymbol{\xi})\ \delta_{N}(x^o - \xi^o)$ for one point object at $x = \xi$ cannot provide the splitting of the four-flow density (6) into the two mirror components, contrary to the operator $\hat{\delta}^4_{N}(x,\xi)_{x\neq\xi}$ for mirror cone-particles. It is in principle impossible to consider two mirror point charges in one reference point ξ because the mirror particles carry opposite charges, $m_{N1} = -m_{N2}$ and $q_{N1} = -q_{N2}$.

Varying (3a) with respect to $\delta P_{N\nu}(x)$, with $\delta g^{\mu\nu}_{N} = 0$, one may reintroduce the Maxwell-type equation (5) in the following operator form,

$$\nabla_\mu g^{\mu\rho}_{N} g^{\nu\lambda}_{N} \left[\frac{d\xi_\lambda[p]}{dp}\frac{\partial\eta_{N}}{\partial x^\rho} - \frac{d\xi_\rho[p]}{dp}\frac{\partial\eta_{N}}{\partial x^\lambda}\right]_{x\neq\xi[p]} = 4\pi\frac{dx^\nu}{dp}\frac{\hat{\delta}^4_{N}(x,\xi[p])_{x\neq\xi[p]}}{\sqrt{-g_{N}}}. \qquad (8)$$

The Euler-Lagrange equation (5) or (8) suggests a way to speculate about the structure of the vector basic cone-field $a_{N\mu}(x)^{s=0}_{x\neq\xi}$ or the scalar basic operator $\eta_{N}(x,\xi[p])_{x\neq\xi}$ in any curved four-space. Both these equations can be easily solved in flat four-space via Green's function, $G(x,x')_{x\neq x'}$=$\{\delta(x^o - x'^o \mp |\mathbf{x}-\mathbf{x}'|)/|\mathbf{x}-\mathbf{x}'|\}_{x\neq x'}$, associated with the fundamental operator equation $\partial_\mu\partial^\mu G(x,x')_{x\neq x'} = 4\pi\hat{\delta}^4(x,x')_{x\neq x'} \equiv 4\pi\{\hat{\delta}^3(\mathbf{x},\mathbf{x}')\delta(x^o - x'^o \mp |\mathbf{x}-\mathbf{x}'|)\}_{x\neq x'}$ (Appendix 1).

Every considered point $\mathbf{x}$ with three-coordinates x^i can be related to sources of different material cones by zero-interval conditions. In other words, different material cone objects can cross one common point $\mathbf{x} \equiv \mathbf{x}_1, \mathbf{x}_2 ..., \mathbf{x}_{N}, ...$ like light or gravity of distant stars cross the Earth at any fixed time. Superposition of different elementary cone objects in one common three-dimensional space $\mathbf{x}$ may be described under the common time interval dt, because all one dimensional intervals (7), $dt^2_{K} \equiv \gamma^{K}_{oo} dx^o_{K} dx^o_{K}$, are flat ($\gamma^{K}_{oo} = \delta_{oo}$ is independent from values of the individual path parameters τ_{K}). We shall prove below that all proper three-spaces x^i_{K}, associated with different objects K, have the universal metric tensor, $\gamma^{K}_{ij} = \delta_{ij}$, and flat intervals, $dl^2_{K} = \gamma^{K}_{ij} dx^i_{K} dx^j_{K}$, contrary to proper four-spaces $x^\mu_{K} = \{x^o_{K}; x^i_{K}\}$ with different metric tensors $g^{K}_{\mu\nu} \neq \eta_{\mu\nu}$ and curves intervals $ds^2_{K} = g^{K}_{\mu\nu} dx^\mu_{K} dx^\nu_{K}$.

For this reason only both the common three-space $\mathbf{x}$, with flat three-interval, and the common flat interval $dt(\mathbf{x})$ are appropriate to apply to all objects, rather than proper four-spaces $x^\mu_{_K}$ (unspecified for the ensemble). Due to the common space+time existence, one may sum (5) over an ensemble of different potential states in $\{\mathbf{x}, dt(\mathbf{x})\}$ and find the following equations for the total four-flow density $i^\nu(x)$ of all intersecting at x cone-particles

$$i^\nu(\mathbf{x},t) \equiv n_o(\mathbf{x})\frac{dx^\nu}{dt} \equiv \sum_{_N} i^\nu_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]} = \frac{\sum_{_N}\nabla_\mu f^{\mu\nu}_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}}{4\pi}, \tag{9}$$

for the four-current density of their gravitomechanical cone-charges (masses)

$$j^\nu_m(\mathbf{x},t) \equiv \mu_o(\mathbf{x})\frac{dx^\nu}{dt} \equiv \sum_{_N} m_{_N} i^\nu_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}$$
$$= \frac{\sum_{_N}\nabla_\mu m_{_N} f^{\mu\nu}_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}}{4\pi} \equiv \frac{\sum_{_N}\nabla_\mu \mathcal{M}^{\mu\nu}_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}}{4\pi}, \tag{10}$$

and for the four-current density of their electric cone-charges

$$j^\nu_q(\mathbf{x},t) \equiv \rho_o(\mathbf{x})\frac{dx^\nu}{dt} \equiv \sum_{_N} q_{_N} i^\nu_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}$$
$$= \frac{\sum_{_N}\nabla_\mu q_{_N} f^{\mu\nu}_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}}{4\pi} \equiv \frac{\sum_{_N}\nabla_\mu \mathcal{F}^{\mu\nu}_{_N}(x)^{s_{_N}[\tau_N]=0}_{x\neq\xi_N[\tau_N]}}{4\pi} \equiv \frac{\nabla_\mu F^{\mu\nu}(\mathbf{x},t)}{4\pi}. \tag{11}$$

The three-space functions $n_o(\mathbf{x}) \equiv \sum_{_N}\{\hat\delta^3(\mathbf{x},\boldsymbol{\xi}_{_N}[\tau_{_N}])/\sqrt{-g}\}^{s_{_N}=0}_{\mathbf{x}\neq\boldsymbol{\xi}_{_N}}$, $\mu_o(\mathbf{x}) \equiv \sum_{_N}\{m_{_N}\hat\delta^3(\mathbf{x},\boldsymbol{\xi}_{_N}[\tau_{_N}])/\sqrt{-g}\}^{s_{_N}=0}_{\mathbf{x}\neq\boldsymbol{\xi}_{_N}}$, $\rho_o(\mathbf{x}) \equiv \sum_{_N}\{q_{_N}\hat\delta^3(\mathbf{x},\boldsymbol{\xi}_{_N}[\tau_{_N}])/\sqrt{-g}\}^{s_{_N}=0}_{\mathbf{x}\neq\boldsymbol{\xi}_{_N}}$ were introduced in (9)-(11) in order to represent the particle matter density, the gravitomechanical charge density, and the electric charge density, respectively, in three-space $\mathbf{x}$ for an ensemble of material cone objects. One will recall, that (9)-(11) were derived for partial kind of motion of each elementary object K, when $W_{_K\mu\nu}(x_{_K}) = 0$.

Note that (11) coincides formally with the Maxwell-Lorentz equation for the electric current density. But the equations (9)-(11) were obtained for infinite cone-particles and cone-fields in space+time, rather than for point particle-sources and cone-fields. The physical densities $n_o(\mathbf{x})$, $\mu_o(\mathbf{x})$, and $\rho_o(\mathbf{x})$, for example, are associated with field cone objects rather than with point objects. In other words the non-dualistic equation (11), for example, relates the continuous field density $j^\nu_q(\mathbf{x},t)$ of cone-charges with the field density $\nabla_\mu F^{\mu\nu}(\mathbf{x},t)$ at every local point of the space+time manifold.

In turn, the electric current density $j^\nu_q(\mathbf{x},t)$, which is specified for cone-charges excluding the source peculiarities, might be formally conjugated to a sum of moving point charge-sources, $\hat q_{_N}$, distributed over these peculiarities. But contrary to the density of the material continuum, the density of the point sources at one fixed point x is meaningless. One should not neglect this obvious fact by trying to formulate a self-consistent theory in the classical way of point carriers of electric charge or mass. One may operate at least with a finite number of peculiarities within a finite volume rather than within a single point.

The requirement for a finite physical magnitude at all space points, for all material objects additionally motivate us to take into consideration an elementary electric charge (and mass) in terms of an elementary continuum (cone) with one reference point (source in cone's vertex) and with a homogeneous charge density $q_{_N}$ (and $m_{_N}$) at all points of this elementary continuum. It seems very unlikely that it is possible to overcome the problem of divergence in classical electrodynamics without changing the accepted paradigm of point charges (but not point sources for the extended charges). "A coherent field theory," stated Einstein (translation [11]), "requires that all its elements be continuous ... And from this requirement arises the fact that the material particle has no place as a basic concept in a field theory. Thus, even apart from the fact that it does not include gravitation, Maxwell's theory cannot be considered a complete theory." The above introduced separation of the extended material particle and its point source (*i.e.* the transformation of the classical point particle into the charged cone continuum) is simply a probe way to complete the theory and to include gravitation into electrodynamics.

An independent equation for the elementary cone-field in its curved proper four-space $x = x_{_N}$,

$$\Big\{\partial_\mu f_{_N\,\nu\delta}(x) + \partial_\nu f_{_N\,\delta\mu}(x) + \partial_\delta f_{_N\,\mu\nu}(x)\Big\}^{s[\tau]=0}_{x\neq\xi[\tau]} = 0, \tag{12}$$

follows directly from the definition of the elementary tensor $f_{_N\,\mu\nu}(x)^{s[\tau]=0}_{x\neq\xi[\tau]}$.

The similar Maxwell-type equation in common space+time,

$$\partial_\mu F_{\nu\delta}(\mathbf{x},t) + \partial_\nu F_{\delta\mu}(\mathbf{x},t) + \partial_\delta F_{\mu\nu}(\mathbf{x},t) = 0, \tag{13}$$

may be considered for the total field tensor $F_{\mu\nu}(\mathbf{x},t) \equiv \sum_K q_{_K} f_{_K\mu\nu}(x)^{s_K[\tau_K]=0}_{x\neq\xi_K[\tau_K]}$.

Notice that the dynamical equation (5) is independent of $m_{_N}$ and $q_{_N}$. The elementary uncharged field with the forming-up four-potential $a_{_N\mu}(x)^{s=0}_{x\neq\xi}$ in this equation may be used as a unified basis for the generation of both the gravitational (Newton) and the electromagnetic (Coulomb) cone-fields.

4. General motion and potential states

4.1. Wave equations

One could formally divide the canonical four-momentum density $P_{_N\mu\nu}(x)^{s=0}_{x\neq\xi}$ of the elementary particle into a gravitomechanical part and an electrical one. Then the canonical tensor $W_{_N\mu\nu}(x)^{s=0}_{x\neq\xi}$ would be formally divided into a gravitomechanical part (with $m_{_N}$) and an electric part (with $q_{_N}$),

$$W_{_N\mu\nu}(x)^{s=0}_{x\neq\xi} = \{m_{_N} M_{\mu\nu}(x) + q_{_N} F^{\neq N}_{\mu\nu}(x)\}^{s=0}_{x\neq\xi}, \tag{14}$$

where $M_{\mu\nu}(x) \equiv [\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)] = w_{\mu\nu} + G^{\neq N}_{\mu\nu}$, $w_{\mu\nu} \equiv [\partial_\mu \mathcal{V}_\nu(x) - \partial_\nu \mathcal{V}_\mu(x)]$, $\mathcal{V}_\nu(x) \equiv \{\beta^{-1}, -\beta^{-1}v_i\}$, $\beta = \sqrt{1 - v_i v^i}$, $G^{\neq N}_{\mu\nu} \equiv [\partial_\mu B^{\neq N}_\nu(x) - \partial_\nu B^{\neq N}_\mu(x)]$, and $F^{\neq N}_{\mu\nu}(x) \equiv [\partial_\mu A^{\neq N}_\nu(x)$ - $\partial_\nu A^{\neq N}_\mu(x)] = \sum^{K\neq N}_K q_{_K}\ [\partial_\mu a_{_K\,\nu}(x) - \partial_\nu a_{_K\,\mu}(x)]^{s_K=0}_{x\neq\xi_K}$, and $x = x_{_N}$. By replacing here the partial derivatives with the covariant ones, we used symmetry of Christoffel coefficients exclusively for the proper four-vector $P_{_N\mu}$ in curved four-space $x^\mu_{_N}$. External fields $a_{_K\mu}$ are not four-vectors in the

proper four-space $x^\mu_{_N}$, and $\nabla_\mu A^{\neq_N}_\nu(x) - \nabla_\nu A^{\neq_N}_\mu(x) \neq \partial_\mu A^{\neq_N}_\nu(x) - \partial_\nu A^{\neq_N}_\mu(x)$, for example. For this reason the gravitomechanical and electric parts of the tensor (14) can not be considered separately as tensors.

The action (3b) for real object may be varied with respect to its proper covariant field under arbitrary material or virtual variations $\delta a_{_N\nu}(x)$. This provides a wave Euler-Lagrange equation for general motion of cone objects in external fields,

$$\nabla_\mu W^{\mu\nu}_{_N}(x)^{s_N[\tau_N]=0}_{x\neq\xi_N[\tau_N]} = 0, \tag{15}$$

where $W^{\mu\nu}_{_N}(x) \equiv g^{\mu\rho}_{_N}(x) g^{\nu\lambda}_{_N}(x) W_{_N\rho\lambda}(x) = -W^{\nu\mu}_{_N}(x)$ and $x = x_{_N}$.

The other independent equation for the canonical tensor density follows from its definition,

$$\{\partial_\mu W_{_N\nu\delta}(x) + \partial_\nu W_{_N\delta\mu}(x) + \partial_\delta W_{_N\mu\nu}(x)\}^{s=0}_{x\neq\xi} = 0 \tag{16}$$

and, when $q_{_N} F^{\neq_N}_{\mu\nu}(x) = 0$,

$$\{m_{_N}[\partial_\mu M_{\nu\delta}(x) + \partial_\nu M_{\delta\mu}(x) + \partial_\delta M_{\mu\nu}(x)]\}^{s=0}_{x\neq\xi} = 0. \tag{17}$$

Now we consider the wave equations (15)-(16) for the canonical tensor density (14) in more detail. The components of the density $F^{\neq_N}_{\mu\nu}(x)$ can be associated with three-vector fields, electric $E_i(x) \equiv F^{\neq_N}_{oi}(x)$ and magnetic $B^i_e(x) \equiv -e^{ijk} F^{\neq_N}_{jk}(x)/2\sqrt{\gamma}$ ($e^{123} = 1$) ones, acting on a cone-particle N with the homogeneous electric charge density $q_{_N}(x) = q_{_N}$. The components of the introduced set of sixteen values $w_{\mu\nu}$ may be associated with the following three-vectors

$$\begin{cases} g_i(x) \equiv \partial_o \mathcal{V}_i - \partial_i \mathcal{V}_o \equiv -\partial_o v_i \beta^{-1} - \partial_i \beta^{-1} \\ h^i(x) \equiv -\frac{e^{ijk}}{2\sqrt{\gamma}}(\partial_j \mathcal{V}_k - \partial_k \mathcal{V}_j) \equiv \{curl \beta^{-1}\mathbf{v}\}^i. \end{cases} \tag{18}$$

Below we shall proof that $\sqrt{\gamma}$ and β are independent from metrics and (18) holds only "flat" components, which are responsible for pure mechanical motion. Gravitational fields are implemented into the proper tensor (14) through the external potentials $B^{\neq_N}_\mu$. According to (1) only these potentials are related to changes of the metric tensor of electrically neutral objects, for which external three-vector fields looks as follows,

$$\begin{cases} G_i(x) \equiv G^{\neq_N}_{oi}(x) \equiv \partial_o B^{\neq_N}_i - \partial_i B^{\neq_N}_o = -\partial_o \beta^{-1}\sqrt{g_{oo}} g_i - \partial_i \beta^{-1}(\sqrt{g_{oo}} - 1) \\ H^i(x) \equiv -\frac{e^{ijk}}{2\sqrt{\gamma}} G^{\neq_N}_{jk}(x) \equiv -\frac{e^{ijk}}{2\sqrt{\gamma}}(\partial_j B^{\neq_N}_k - \partial_k B^{\neq_N}_j) = \{curl\ \beta^{-1}\sqrt{g_{oo}}\mathbf{g}\}^i. \end{cases} \tag{19}$$

Now the equation (17) for pure gravitomechanical systems may be read in a three-vector form,

$$\{m_{_N} div\ (\mathbf{h} + \mathbf{H})\}^{s=o}_{x\neq\xi} = 0 \tag{20}$$

and

$$\{m_{_N}[\{curl\ (\mathbf{g} + \mathbf{G})\}^i + \partial_o(h^i + H^i)]\}^{s=o}_{x\neq\xi} = 0, \tag{21}$$

because of equalities $div\ curl\ \mathbf{a} = 0$ and $curl\ grad\ \mathbf{a} = 0$ for $\{curl\ \mathbf{a}\}^i \equiv (2\sqrt{\gamma})^{-1} e^{ijk}(\partial_j a_k - \partial_k a_j)$, $div\ \mathbf{a} \equiv \gamma^{-1/2}\partial_i(\sqrt{\gamma} a^i)$.

One can also represent the general equation (16) at real field points, $x \neq \xi[\tau]$ and $s(x, \xi[\tau]) = 0$, of electrically charged objects in a three-vector form,

$$div[m_{_N}\mathbf{h} + m_{_N}\mathbf{H} + q_{_N}\mathbf{B}]^{s=o}_{x\neq\xi} = 0, \tag{22}$$

$$\{(curl[m_{_N}\mathbf{g}+m_{_N}\mathbf{G}+q_{_N}\mathbf{E}])^i+\partial_o[m_{_N}h^i+m_{_N}H^i+q_{_N}B^i]\}^{s=o}_{x\neq\xi}=0. \quad (23)$$

The difference between (20)-(21) and (22)-(23) is related with the different proper spaces $x = x_{_N}$ for charged and uncharged masses. (Below we shall find that pseudo-Riemannian metrics of charged objects is associated with external electromagnetic potentials, and one should use in (19) replacements $G_i \to G_i + (q_{_N}/m_{_N})E_i$, $H^i \to H^i + (q_{_N}/m_{_N})B^i$ for relations with the similar components of the novel metric tensor).

Contrary to classical theory, which admits bulk (free of particles) three-space regions, the equation (9), for example, can not be applied with zero density of particle matter at any point $\mathbf{x}$. Charged cone matter of the same elementary particle-field object is emitted from different positions of its source and this elementary matter crosses all different three-space points $\mathbf{x}$ at the same time parameter. The elementary cone-particle (and cone-charge) density exists simultaneously at all three-space points (the same is true for the elementary cone-field density). For these reasons, a total superposition of cone-particles (and cone-charges) always has to be present at any three-space point, *i.e.* $n_o(\mathbf{x}) \neq 0$ and $\mu_o(\mathbf{x}) \neq 0$ for all $\mathbf{x}$ (while $\rho_o(\mathbf{x})$ could be equal to zero at some three-space points only due to the opposite signs of the electric charge densities in the material superposition). Three-space is actually a space-charge-mass continuum without bulk regions, and source peculiarities are not included in this material medium.

4.2. Superfluid or potential states

To apply the derived equations to practical problems of condensed matter, for example, we consider the simplest partial solutions of (15) and (16) in the absence of the field $P_{_N\mu}$ vorticity, $W_{_N\mu\nu}(x)^{s=0}_{x\neq\xi} \equiv 0$, when $I^{\mu}_{_N}(x)^{s=0}_{x\neq\xi} \equiv 0$ due to (5). Such potential state, $\partial_\mu P_{_N\nu} = \partial_\nu P_{_N\mu}$, is well known for macroscopic superfluid systems. The canonical four-momentum density $P_{_N\mu}(x)^{s=0}_{x\neq\xi}$ can be written in this case via a scalar potential $\Upsilon_{_N}(x)^{s=0}_{x\neq\xi}$, *i.e.*

$$P_{_N\mu}(x)^{s=0}_{x\neq\xi} \equiv \{m_{_N}(x)u_\mu(x)+q_{_N}(x)A^{\neq_N}_\mu(x)\}^{s=0}_{x\neq\xi} = -\partial_\mu\Upsilon_{_N}(x)^{s=0}_{x\neq\xi}, \quad (24)$$

with $\partial_\mu\partial_\nu\Upsilon_{_N}(x)^{s=0}_{x\neq\xi} = \partial_\nu\partial_\mu\Upsilon_{_N}(x)^{s=0}_{x\neq\xi}$. The potential state, $W_{_N\mu\nu}(x)^{s[\tau]=0}_{x\neq\xi[\tau]} = 0$, corresponds to superfluid matter without radiation or energy exchange and reads at all points x of the elementary cone N in terms of the three-vector functions

$$\{m_{_N}g_i(x)+m_{_N}G_i(x)+q_{_N}E_i(x)\}^{s[\tau]=0}_{x\neq\xi[\tau]}=0, \quad (25)$$

$$\{m_{_N}h^i(x)+m_{_N}H^i(x)++q_{_N}B^i(x)\}^{s[\tau]=0}_{x\neq\xi[\tau]}=0. \quad (26)$$

These dynamic equations describe the mutual counterbalance of gravitomechanical and electromagnetic forces acting on the cone-charge densities $m_{_N}$ and $q_{_N}$ in the presence of external electromagnetic field and gravity. The equations (25)-(26) are not new for dissipationless macroscopic systems. Actually (25) is a relativistic generalization of the Bernoulli stationary equation for a

charged ideal fluid, while (26) exhibits the known fact that the London stationary supercurrent is proportional to an electromagnetic three-vector potential in a superconductor.

The potential states, which are associated with equations (24)-(26), may be applied to many known problems, such as the field description of the Cooper pairs within a superconductor, the dissipationless motion of free electrons within a conductor, the bound electron states in atoms, etc. Each of these elementary carriers, which are not involved in collisions, can be characterized by the individual potentials $\Upsilon_{_K}(x)^{s=0}_{x\neq\xi}$ (corresponding to the phases of wavefunctions for elementary matter in quantum theory). The potential state of the extended charged object, when $P_{_N\mu}(x)^{s=0}_{x\neq\xi} = -\partial_\mu \Upsilon_{_N}(x)^{s=0}_{x\neq\xi}$ at all points of the proper four-space $x = x_{_N}$, may be considered as a superfluid state of this cone-object in the common space+time $\{\mathbf{x}, dt\}$.

Now we shall consider a uniformly rotating conductor (the Faraday disk) in order to analyze the relativistic experiment of Ref. [14] in terms of the approach, developed herein. Rotation leads to the gravito-inertial fields (18) and (19). The induced stationary electric and magnetic fields within the conductor create compensating Lorentz forces that allow free charges to rotate synchronously with the lattice charges. Only a small (conducting) fraction of electrons on the Fermi surface deviates from potential states due to inelastic collisions. One may say that stationary charged cone-particles (electrons within the Fermi volume) take potential states and satisfy (24) - (26).

We find the electric and magnetic fields within a uniformly rotating conductor with an angular frequency $\boldsymbol{\omega}$ in an inertial frame (where $< \mathbf{v}_{_K} >_{_K} = \boldsymbol{\omega} \times \mathbf{r}$, $G_i \to 0, H^i \to 0$) by averaging (25) and (26), respectively, over the ensemble,

$$< \mathbf{E}_q(x) >_{_K} = < \boldsymbol{\partial} \frac{m_{_K}}{q_{_K}\sqrt{1-\mathbf{v}^2_{_K}}} >_{_K} \approx -\frac{m_o \omega^2 \mathbf{r}}{|q_o|(1- < v^2_{_K} >_{_K})^{3/2}}, \tag{27}$$

$$< \mathbf{B}(x) >_{_K} = - < \frac{m_{_K}}{q_{_K}} curl \ \frac{\mathbf{v}_{_K}}{\sqrt{1-\mathbf{v}^2_{_K}}} >_{_K} \approx \frac{2 m_o \boldsymbol{\omega}}{|q_o|\sqrt{1- < v^2_{_K} >_{_K}}}, \tag{28}$$

where $q_{_K} = -|q_o| < 0$ is the negative electron charge, $m_{_K} = m_o$ is the rest electron mass, and $m_o(1- < v^2_{_K} >_{_K})^{-1/2}$ is the relativistic electron mass averaged over the Fermi volume ($0 \le v^2_{_K} \le v^2_{_F} \approx 10^{-4}$, where $v_{_F}$ is the Fermi velocity).

The electric and magnetic fields within a uniformly rotating superconductor can also be determined from (27) and (28), respectively, by averaging over all free electrons of the total Fermi volume and Cooper pairs: both normal and paired electrons are in potential states. Relatively small fractions of paired and conducting electrons (both fractions on the Fermi surface) may provide, in our consideration, only relatively small contributions to the relativistic corrections in (28).

By using the relativistic accurate data of the experiment [14] for the magnetic flux within rotating niobium superconductors (for which $(1-v^2_{_F})^{-1/2} = 1.000180$ due to the independent Fermi surface data), we may conclude for the mass-charge ratio from (28) for this experiment that $1.000084(21) m_o/q_o = m/|q| = m_o/(1- < v^2_{_K} >_{_K})^{1/2}|q_o|$. This result, *i.e.* $< v^2_{_K} >_{_K} = 0.47(\pm 12) v^2_{_F}$, confirms the above statement about a dominant contribution to the London magnetic moment from dissipationless electron states under the Fermi surface, rather

than only from the Cooper pairs on the Fermi surface (when one might expect $< v_{_K}^2 >_{_K} = v_{_F}^2$).

Conventional theory predicts that superfluid fractions of free electrons has to take only Fermi velocities, $v_{_K} = v_{_F}$, that evidently disagrees with the experimental results. The experiment [14] demonstrated, that the potential states of charged fermions within the Fermi volume is also a superfluid form of matter. But only superfluid Cooper bosons are responsible for the observed quantization of the London magnetic moment due to the non-vanishing macroscopic potential $< \Upsilon_{_K}^s >_{_K} = \Upsilon_{_K}^s \neq 0$. Each fermion possesses a stationary superfluid state with an individual potential $\Upsilon_{_K}^n$. However, any ensemble of fermions cannot exhibit macroscopic superfluid properties in space+time because $< \Upsilon_{_K}^n >_{_K} = 0$ after averaging over the Fermi volume, rather than over the equipotential Fermi surface in the case of the paired electrons. Nevertheless the Barnet magnetic moment of rotating normal metals is also associated with superfluid states of fermions without the moment quantization on the macroscopic level.

Again, the developed approach to free electrons in potential states can explain the measured relativistic mass-charge ratio of superfluid carriers that is hitherto unexplained by the currently available theories.

Superfluid states of charges, when $W_{_N\mu\nu}(x)^{s=0}_{x\neq\xi} \equiv 0$ and $I^\mu_{_N}(x)^{s=0}_{x\neq\xi} \equiv 0$ (and $T^{\mu\nu}_{_N} = 0$, introduced below), are very special cases of matter motion without energy exchange and radiation. Below we consider more general forms of motion for which the incomplete equation (5) cannot be applied.

5. Proper energy-tensor of the extended object

The Hilbert variation procedure [19] for (3a) with respect to variation of the proper metric tensor, $\delta g_{\mu\nu}(x) \equiv \delta g^{_N}_{\mu\nu}(x)$ should provide the symmetric energy-tensor density, $T^{\mu\nu}_{_N}(x)^{s=o}_{x\neq\xi}$, of the elementary particle-field object N. One may fix under this variation the contravariant coordinate vectors dx^μ (but not the covariant ones, $\delta dx_\nu = \delta(g_{\mu\nu}dx^\mu) = dx^\mu \delta g_{\mu\nu}$), the universal scalars $m_{_N}(x)$ and $q_{_N}(x)$, the covariant four-vector potential $a_{_N\nu}(x)^{s=0}_{x\neq\xi}$ and the covariant field tensor $f_{_N\mu\nu}(x)^{s=0}_{x\neq\xi}$. Note that symmetric components of $g_{\mu\nu}$ are not independent one from another, $\delta g_{\mu\nu} = \delta g_{\nu\mu}$, and we define $\delta S \equiv -\int dx^4\sqrt{-g}(T^{\mu\nu}\delta g_{\mu\nu} + T^{\nu\mu}\delta g_{\nu\mu})/2 = -\int dx^4\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu}$.

The proper canonical four-momentum $P_{_N\mu}$ depends on external gravitational and electromagnetic fields (the scheme with $P_{_N\mu} = m_{_N} g^{_N}_{\mu\nu} dx^\mu/ds$ in the proper canonical four-space will be clarified below). Its variations are not independent from the variations of the metric tensor, $i^\mu \delta P_\mu = m_{_N} i^\mu \delta[g_{\mu\nu}dx^\nu(g_{\rho\lambda}dx^\rho dx^\lambda)^{-1/2}]$ $= [(\delta g_{\mu\nu})m_{_N} i^\mu dx^\nu/2ds] + [(\delta g_{\nu\mu})m_{_N} i^\nu dx^\mu/2ds] = m_{_N}(i^\mu dx^\nu + i^\nu dx^\mu)\delta g_{\mu\nu}/2ds$. The term $-\sqrt{-g}(f^{\mu\nu}_{_N}\delta W_{_N\mu\nu} + f^{\nu\mu}_{_N}\delta W_{_N\nu\mu})/16\pi$ may be transformed under the four-space integral into $(m_{_N}\sqrt{-g}\nabla_\nu f^{\nu\mu}_{_N}/4\pi)\delta P_{_N\mu}$. The contravariant metric tensor is related to the covariant one, *i.e.* $\delta g^{\alpha\beta} = -g^{\alpha\mu}g^{\beta\nu}\delta g_{\mu\nu} - g^{\alpha\nu}g^{\beta\mu}\delta g_{\nu\mu}$ $= -\delta g_{\mu\nu}(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu})$, $\delta\sqrt{-g} = \sqrt{-g}(g^{\mu\nu}\delta g_{\mu\nu} + g^{\nu\mu}\delta g_{\nu\mu})/2 = \sqrt{-g}(g^{\mu\nu} + g^{\nu\mu})\delta g_{\mu\nu}/2$.

There are no special reasons in our approach to involve artificially a scalar metric curvature, R_{Ricci}, into the complete action (3a) for the collisionless particle-field object. The curvature ought to appear naturally in any self-contained theory. Moreover, the Rainich - Misner criterion, $R_{_{RM}}$=0, for unified theories [20,21] dismisses scalar curvatures in the initial dynamical equations.

Finally, after variation (3a) with respect to $\delta g^{N}_{\mu\nu}$ (and $\delta g^{N}_{\nu\mu}$), one can obtain the proper energy-tensor density,

$$T^{\mu\nu}_{N}(x)^{s[\tau]=0}_{x\neq\xi[\tau]} \equiv \{\frac{m_N}{2}\left(\frac{dx^\mu}{ds_N}I^\nu_N(x)+\frac{dx^\nu}{ds_N}I^\mu_N(x)\right)$$
$$+\frac{W_{N\rho\lambda}(x)}{16\pi}[g^{\mu\nu}_N f^{\rho\lambda}_N(x)-2g^{\mu\rho}_N f^{\nu\lambda}_N(x)-2g^{\nu\rho}_N f^{\mu\lambda}_N(x)]\}^{s[\tau]=0}_{x\neq\xi[\tau]}, \quad (29)$$

for one elementary particle-field object N at its material points $x_N = x$, with $x \neq \xi[\tau]$ and $s(x,\xi[\tau]) = 0$. This tensor takes only zero components for the above considered potential motion, when $I^\mu_N = W_{N\mu\nu} = 0$. One may say that the paired in the selected object cone-particle and cone-field fractions energetically compensate (or screen) each other for these dissipationless states of matter.

If ten different components of the proper metric tensor $g^N_{\mu\nu}$ can be independent one from another, as is generally accepted, one could expect to derive from the action (3a) the Einstein-type tensor equation,

$$T^{\mu\nu}_{N}(x)^{s[\tau]=0}_{x\neq\xi[\tau]} = 0, \quad (30)$$

for every considered object.

Below we shall derive a novel four-vector equation for arbitrary motion of the particle-field object, $T^{\mu\nu}_N P_{N\mu}$=0, rather than the Einstein-type tensor equation for gravitation, $T^{\mu\nu}_N = 0$, which takes place in vector electrogravity only for potential, superfluid states. By deriving the ten-component Einstein equation from the variational principle, one accepts the conventional assumption of general relativity that components of the symmetric tensor $g_{\mu\nu}$ may have ten degrees of freedom (without referring to any physical notions behind this basic mathematical statement). As a result ten components of the energy-tensor, rather then masses with Newton's vector fields, are origin of tensor gravity in general relativity. In the last section we shall find that the metric tensor or its tetrad can take only four degrees of freedom associated with external Coulomb-Newton four-potentials $a_{K\mu}$. A reasonable four-vector equation for gravitation has to replace ten tensor relations in the most general case of motion.

Total density of matter at any selected point of the common three-space $\mathbf{x}$ takes contribution from different extended cone-objects. In order to describe the energy-tensor density under intersection of traceless superfluid objects at one particular point of space+time and derive the Einstein-type equation, one could consider a sum of the elementary densities,

$$T^{\mu\nu}(\mathbf{x},t) \equiv \sum_N T^{\mu\nu}_N(x_N)^{s_N[\tau_N]=0}_{x_N\neq\xi_N[\tau_N]} = \sum_N 0 = 0. \quad (31)$$

But there are no 4D intersections of different curved four-spaces x_N, only their 3D subspaces with universal geometry can intersect. A transaction in (31) or in (9)-(11) from different proper four-spaces x_N to the common space+time manifold $\{\mathbf{x}; dt(\mathbf{x})\}$ is not a trivial procedure (that will be clarified in the next section).

Assuming for a moment that it is possible to introduce the curved four-space with common (or averaged) metric tensor, $\tilde{g}_{\mu\nu}$, with $d\tilde{s}^2 = \tilde{g}_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu$, one could formally rewrite (31) in the Einstein-like form,

$$\frac{1}{\kappa}\left(\tilde{R}^{\mu\nu}-\frac{\tilde{g}^{\mu\nu}}{2}\tilde{g}_{\rho\lambda}\tilde{R}^{\rho\lambda}\right) = \mu_o(\tilde{\mathbf{x}})\frac{d\tilde{x}^\mu}{d\tilde{x}^o}\frac{d\tilde{x}^\nu}{d\tilde{s}}$$

$$+\frac{\tilde{F}_{\rho\lambda}}{16\pi}[\tilde{g}^{\mu\nu}\tilde{F}^{\rho\lambda} - 2\tilde{g}^{\mu\rho}\tilde{F}^{\nu\lambda} - 2\tilde{g}^{\nu\rho}\tilde{F}^{\mu\lambda}], \tag{32}$$

where the following definitions are introduced, $\tilde{R}^{\mu\nu} \equiv \tilde{G}^{\mu\nu} - 2^{-1}\tilde{g}^{\mu\nu}\tilde{g}_{\rho\lambda}\tilde{G}^{\rho\lambda}$,

$$\tilde{G}^{\mu\nu}(\tilde{x}) \equiv \tilde{G}^{\mu\nu}(\tilde{\mathbf{x}}, \tilde{x}^o) \equiv \frac{\kappa}{8\pi}\sum_N \{m_N \frac{dx^\mu_N}{ds_N}\nabla_\rho f^{\rho\nu}_N(x_N) + m_N \frac{dx^\nu_N}{ds_N}\nabla_\rho f^{\rho\mu}_N(x_N)$$

$$+m_N M_{\rho\lambda}(x_N)[g^{\mu\rho}_N f^{\nu\lambda}_N(x_N) + g^{\nu\rho}_N f^{\mu\lambda}_N(x_N) - \frac{g^{\mu\nu}_N}{2} f^{\rho\lambda}_N(x_N)]\}^{s_N[\tau_N]=0}_{x_N\neq\xi_N[\tau_N]},$$

$$\mu_o(\tilde{\mathbf{x}})\frac{d\tilde{x}^\mu}{d\tilde{x}^o}\frac{d\tilde{x}^\nu}{d\tilde{s}} \equiv \sum_N \frac{m_N}{2}\left(i^\mu_N(x_N)\frac{dx^\nu_N}{ds_N} + i^\nu_N(x_N)\frac{dx^\mu_N}{ds_N}\right)^{s_N[\tau_N]=0}_{x_N\neq\xi_N[\tau_N]},$$

$$\tilde{F}_{\rho\lambda}[\tilde{g}^{\mu\nu}\tilde{F}^{\rho\lambda} - 2\tilde{g}^{\mu\rho}_N\tilde{F}^{\nu\lambda} - 2\tilde{g}^{\nu\rho}\tilde{F}^{\mu\lambda}_N]$$

$$\equiv \sum_N F^{\neq N}_{\rho\lambda}(x_N)[g^{\mu\nu}_N \mathcal{F}^{\rho\lambda}_N(x_N) - 2g^{\mu\rho}_N \mathcal{F}^{\nu\lambda}_N(x_N) - 2g^{\nu\rho}_N \mathcal{F}^{\mu\lambda}_N(x_N)]^{s_N[\tau_N]=0}_{x_N\neq\xi_N[\tau_N]}.$$

A trace of the Einstein-type equation (32), $-\kappa^{-1}\tilde{R} = -\kappa^{-1}\tilde{g}_{\rho\lambda}\tilde{R}^{\rho\lambda}$, with

$$-\frac{\tilde{R}}{\kappa} = \mu_o(\tilde{\mathbf{x}})\frac{d\tilde{s}}{d\tilde{x}^o} \equiv \sum_N m_N \int dp_N \frac{\sqrt{g^N_{\mu\nu}dx^\mu_N dx^\nu_N}}{dp_N}\frac{\hat{\delta}^4_N(x_N, \xi_N[p_N])_{x_N\neq\xi_N}}{\sqrt{-g_N}}, \tag{33}$$

depends on the "curvature" $\tilde{R}(\tilde{x})$ ($k = 8\pi G$ is the Einstein constant [8]) and formally corresponds to the equality $\sum_N g^N_{\mu\nu} T^{\mu\nu}_N(x_N)^{s_N[\tau_N]=0}_{x\neq\xi_N[\tau_N]} \equiv 0$.

The above separation of gravitomechanical and electromagnetic fields in (32) looks artificial for the introduced cone object with joint forming-up field for electric charge and mass densities. It is more reasonable to divide, if ever, (31) into the pure cone-particle and pure cone-field contributions as follows,

$$\frac{1}{\kappa}\tilde{\mathcal{R}}^{\mu\nu}(\tilde{\mathbf{x}}, \tilde{x}^o) = \mu_o(\tilde{\mathbf{x}})\frac{d\tilde{x}^\mu}{d\tilde{x}^o}\frac{d\tilde{x}^\nu}{d\tilde{s}}, \tag{34}$$

where

$$\tilde{\mathcal{R}}^{\mu\nu}(\tilde{x}) \equiv \frac{\kappa}{8\pi}\sum_N \{m_N \frac{dx^\mu_N}{ds_N}\nabla_\rho f^{\rho\nu}_N(x_N) + m_N \frac{dx^\nu_N}{ds_N}\nabla_\rho f^{\rho\mu}_N(x_N)$$

$$+W_{N\rho\lambda}(x_N)[g^{\mu\rho}_N f^{\nu\lambda}_N(x_N) + g^{\nu\rho}_N f^{\mu\lambda}_N(x_N) - \frac{g^{\mu\nu}_N}{2} f^{\rho\lambda}_N(x_N)]\}^{s_N[\tau_N]=0}_{x_N\neq\xi_N[\tau_N]}. \tag{35}$$

Again, the equation (34) looks artificial for extended masses under the "averaged" geometry of the joint curved four-space $\tilde{x}$ and the joint curved three-space $\tilde{\mathbf{x}}$, where intersection of cone-particles can be expected.

The external forming-up fields $a_{K\nu}(x)^{s=0}_{x\neq\xi}$ can be traced under formation of the field densities in (32) and (35), which may be formally associated with the Ricci curvature in the Einstein equation. One may assume that the metric tensor could also depend on contributions of external fields with $a_{K\mu}$ and $a_{K\nu}$. But these forming-up fields are common for both elementary Newton-like, $\mathcal{B}_{N\nu}(x)^{s=0}_{x\neq\xi} \equiv m_N a_{N\nu}(x)^{s=0}_{x\neq\xi}$, and Coulomb-like, $\mathcal{A}_{N\nu}(x)^{s=0}_{x\neq\xi} \equiv q_N a_{N\nu}(x)^{s=0}_{x\neq\xi}$, four-potentials. These particular findings can motivate us to reconsider the

metric properties of the proper four-space $x_{_N}$, that may open the gates for modernization of gravity within Einstein's covariant formalism. Unification for gravitation and electromagnetism may take place, for example, under the canonical four-space with electromagnetic and gravitomechanical connections. Below we shall study the symmetrical involvements of external masses and charges into the proper metric tensor and develop the new covariant approach to the extended cone-particle within the flat 3D continuum of space-charge-mass.

6. Gravitation under flat three-space

The covariant constructions for every selected object depend essentially on proper four-space geometry associated with external matter. External matter (and proper four-space geometry) differs for different objects. Curved four-space in general relativity is associated with curved three-space due to Schwarzschild's solution for point masses [22].

There are no point masses in our consideration and Schwarzschild's solutions cannot be appropriate for extended cone-particles. Nevertheless, should all statements of the accepted gravitomechanics be formally adopted for cone-masses, the above derived field equations would meet the problem of inhomogeneously curved 3D subspaces. It would not make sense to speak about a common three-space for any ensemble of intersecting extended objects. It is impossible to introduce a common pseudo-Riemannian four-space $\tilde{x}$ with $g_{_1\mu\nu}(\tilde{x})p^{\mu}_{_1}(\tilde{x})p^{\nu}_{_1}(\tilde{x}) = m^2_{_1}(\tilde{x})$ and $g_{_2\mu\nu}(\tilde{x})p^{\mu}_{_2}(\tilde{x})p^{\nu}_{_2}(\tilde{x}) = m^2_{_2}(\tilde{x})$ for different extended elementary mass densities at the same points of the same manifold $\tilde{x}$, because $g_{_1\mu\nu}(\tilde{x}) \neq g_{_2\mu\nu}(\tilde{x})$ due to different external systems for $m_{_1}$ and $m_{_2}$ at the same points. But extended intersections of different cone objects may take place on joint subspaces when and if the latter hold common geometry.

Einstein's covariant formalism can fluently operate, as known, with different solutions for three-space metric under the pseudo-Riemannian four-interval. Evolution of extended cone-particles can be observed through the dynamics of their point sources in common material 3D space which should keep one universal geometry for all proper 3D subspaces. Observed conservation of three-momentum for mechanical systems at all space points indicates that the common 3D space is to be homogeneous with constant curvature (positive, negative or zero).

In order to verify mathematical opportunities to implement flat three-space into Einstein's scheme with pseudo-Riemannian metric, we employ the known tetrad formalism, for example [8,23], which leads to the representation of a four-interval, $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \equiv \eta_{\alpha\beta}e^{\alpha}_{\ \mu}e^{\beta}_{\ \nu}dx^{\mu}dx^{\nu} \equiv \eta_{\alpha\beta}dx^{\alpha}dx^{\beta}$, in the "plane" coordinates $dx^{\alpha} \equiv e^{\alpha}_{\ \mu}dx^{\mu}$, $dx^{\beta} \equiv e^{\beta}_{\ \nu}dx^{\nu}$, $\eta_{\alpha\beta} = diag(+1,-1,-1,-1)$. One can immediately find $e^{o}_{\ \mu} = \{\sqrt{g_{oo}}; -\sqrt{g_{oo}}g_i\}$ and $e^{a}_{\ \mu} = \{0, e^{a}_{\ i}\}$ from the equality $ds^2 \equiv [\sqrt{g_{oo}}(dx^o - g_i dx^i)]^2 - \gamma_{ij}dx^i dx^j$. At first glance the space triad $e^{a}_{\ i}$ (a = 1,2,3), which can be algebraically represented via the components of the three-space metric tensor $\gamma_{ij} \equiv g_{oi}g_{oj}g^{-1}_{oo} - g_{ij}$, depends essentially on gravitation fields or four-space distribution of gravitomechanical cone-charges. But this is not the case due to the universal degeneration of the three-space metric tensor γ_{ij} for any elementary object.

Let us consider the "curved" three-space components $V_i \equiv \mathcal{V}_i + B^{\neq N}_i$ of the metric-velocity four-vector $V_{\mu} \equiv g_{\mu\nu}\dot{x}^{\nu}$ in (1) by using the tetrad formalism,

$-(\sqrt{g_{oo}}g_i+v_i)(1-v_iv^i)^{-1/2} = V_i = e^\alpha_{\ i}V_\alpha = e^o_{\ i}V_o+e^a_{\ i}V_a = -(\sqrt{g_{oo}}g_i+e^a_{\ i}v_a)(1-v_av^a)^{-1/2}$. This leads immediately to a trivial solution $v_i \equiv e^a_{\ i}v_a = \delta^a_i v_a$ for the "curved", v_i, and the "plane", v_a, three-velocities, because $e^o_{\ i} = -\sqrt{g_{oo}}g_i$ and $V_\alpha = \{(1-v_av^a)^{-1/2}; -v_a(1-v_av^a)^{-1/2}\}$. The solution with the Kronecker delta-symbol δ^a_i ($\delta^a_i = 1, i = a$ and $\delta^a_i = 0, i \neq a$) demonstrates that the space triad and, consequently, the three-space metric tensor are independent of gravitation fields, *i.e.* $e^a_{\ i} = \delta^a_i$ and $\gamma_{ij} = \delta_{ij}$.

Euclidean three-space geometry may be appropriate for the covariant formalism of gravitation due to the hidden equalities $g_{oi}g_{oj}g^{-1}_{oo} \equiv g_{ij} + \delta_{ij}$ for every elementary material object. The metric tensor in the most general case reads $g_{\mu\nu} \equiv \eta_{\alpha\beta}e^\alpha_{\ \mu}e^\beta_{\ \nu} \equiv \eta_{\mu\nu} + \eta_{\alpha\beta}(e^\alpha_{\ \mu}e^\beta_{\ \nu} - \delta^\alpha_\mu\delta^\beta_\nu) \equiv \eta_{\mu\nu} + g^{\neq N}_{\mu\nu}$, where $e^o_{\ \mu} = \{\sqrt{g_{oo}}; -\sqrt{g_{oo}}g_i\}$ and $e^a_{\ \mu} = \{0, \delta^a_i\} \equiv \delta^a_\mu$. In agreement with this consideration, the three-interval is always associated with the universal Euclidean metric, because $\gamma^N_{ij} \equiv \{g_{oi}g_{oj}g^{-1}_{oo} - g_{ij}\}^N \equiv \delta_{ij} \equiv -\eta_{ij}$ for all objects, while the four-interval is always associated with the proper pseudo-Riemannian metric, $g_{\mu\nu} \neq \eta_{\mu\nu}$, which is different for different elementary objects.

A scalar differential of the four-interval along material points $x \equiv x_N$ of any selected cone object N (four-interval $ds_N \equiv ds$, for brevity) is given by

$$ds^2 = \left[\sqrt{g_{oo}}\frac{dx^o}{ds} + \frac{g_{oi}dx^i}{\sqrt{g_{oo}}ds}\right]^2 ds^2 - \delta_{ij}dx^idx^j \equiv d\tau^2(ds,dt,dl) - dl^2 \qquad (36)$$

in arbitrary external gravitational fields. But (36) is a nonlinear equation with respect to ds^2, rather than a linear relation. The first term at the right hand side of (36) depends on the four-interval ds, which is a nonlinear function of the 3D interval $dl \equiv dl_N \equiv \sqrt{\delta_{ij}dx^idx^j}$ and 1D interval $dt \equiv dt_N \equiv \sqrt{\delta_{oo}dx^odx^o}$.

Now we return to the metric-velocity four-vector in (1). Notice that $V_\mu = e^\alpha_{\ \mu}V_\alpha = (e^a_{\ \mu}V_a + e^o_{\ \mu}V_o) = (e^a_{\ \mu}V_a + \delta^o_\mu V_o) + (e^o_{\ \mu} - \delta^o_\mu)V_o \equiv \mathcal{V}_\mu + B^{\neq N}_\mu$, with the proper four-velocity $\mathcal{V}_\mu \equiv (e^a_{\ \mu}V_a + \delta^o_\mu V_o) = \delta^\alpha_\mu V_\alpha$, because $e^a_{\ o} = 0$ and $e^a_{\ i} = \delta^a_i$. Flat three-space geometry is just the way to introduce the four-potential $B^{\neq N}_\mu \equiv (e^o_{\ \mu} - \delta^o_\mu)V_o$ of external gravitational field in analogy with the four-potential $A^{\neq N}_\mu$ for external electromagnetic field. This external gravitational four-potential and the proper metric tensor $g_{\mu\nu} \equiv g^N_{\mu\nu}$ are characteristics of only one selected object N. In general the four-momentum density, $P_{N\mu}(x)^{s=o}_{x\neq\xi} \equiv m_N\delta^\alpha_\mu V_\alpha + m_N B^{\neq N}_\mu + q_N A^{\neq N}_\mu$, takes the mechanical, gravitational and electromagnetic contributions, respectively. Both gravitational and electromagnetic contributions are associated with the same system of external forming-up fields $a_{K\mu}$, that provides for a neutral object, $q_N = 0$, the following relations,

$$p_{N\mu}(x) \equiv \left\{\frac{m_N}{\sqrt{1-v^2}} + \frac{m_N(\sqrt{g_{oo}}-1)}{\sqrt{1-v^2}}; -\frac{m_N v_i}{\sqrt{1-v^2}} - \frac{m_N g_i\sqrt{g_{oo}}}{\sqrt{1-v^2}}\right\}$$
$$\equiv m_N\delta^\alpha_\mu V_\alpha + \sum_K^{K\neq N}(-Gm_Nm_K)a_{K\mu}(x)^{s_K=o}_{x\neq x_K}. \qquad (37)$$

At the right hand side we used the symmetrical involvement of any mass, m_K, and electric charge, q_K, in their proper gravitational and electromagnetic field, based on the joint forming-up uncharged field $a_{K\mu}$. This principle statement of the developed scheme makes external gravitational field linear with respect to the sources and provides new opportunities to introduce a detail structure

of the metric tensor. Both gravitational, $B^{\neq N}_\mu(x) \equiv -G\sum_K^{K\neq N} m_K a_{K\mu}(x)^{s_K=o}_{x\neq\xi_K}$, and electromagnetic, $A^{\neq N}_\mu(x) \equiv \sum_K^{K\neq N} q_K a_{K\mu}(x)^{s_K=o}_{x\neq\xi_K}$, four-potentials may lead to the joint gauge-invariant scheme and to conservations of the charges, m_N and q_N, respectively. One will recall that the mechanical and the gravitational charges in (37) are equal.

In this section we study only neutral objects ($q_N = 0$ for the selected object N) by staying in the framework of the mechanical covariant formalism. Then the tetrad takes, according to (37), the following components $e^a{}_\mu = \{0, \delta^a_i\} = \delta^a_\mu$ and $e^o{}_\mu = \{1+\sqrt{1-v^2}B^{\neq N}_o; \sqrt{1-v^2}B^{\neq N}_i\} = \delta^o_\mu + \sqrt{1-v^2}B^{\neq N}_\mu$, and the proper metric tensor $g^N_{\mu\nu} \equiv \eta_{\mu\nu} + g^{\neq N}_{\mu\nu}$ for the elementary cone-charge m_N in external fields is given by

$$\begin{cases} g_{oo} = (1+\sqrt{1-v^2}B^{\neq N}_o)^2 \\ g_{oi} = (1+\sqrt{1-v^2}B^{\neq N}_o)\sqrt{1-v^2}B^{\neq N}_i \\ g_{ij} = (1-v^2)B^{\neq N}_i B^{\neq N}_j + \eta_{ij} \end{cases} . \tag{38}$$

The considered point x of the selected object N is affected by all other objects K with retarded zero-interval signals from their source point $\xi_K(\tau_K)$. As expected, all components of the three-space metric tensor are independent of external gravitational charges, $\gamma_{ij} \equiv g_{oi}g_{oj}g^{-1}_{oo} - g_{ij} = \delta_{ij}$ (now may be verified from (38)), while every particular component of the proper four-space metric tensor (38) is related to the external potential $B^{\neq N}_\mu(x)$.

One can represent (38) in a more compact way, $g_{\mu\nu} = \eta_{\alpha\beta}e^\alpha{}_\mu e^\beta{}_\nu$ and $e^\alpha{}_\mu = \delta^\alpha_\mu + \delta^{\alpha o}\sqrt{1-v^2}B^{\neq N}_\mu$. Notice, that $P^\mu_N = g^{\mu\nu}P_{N\nu} = m_N[\eta^{\mu\nu}\mathcal{V}_\nu + (g^{\mu\nu}-\eta^{\mu\nu})\mathcal{V}_\nu + g^{\mu\nu}B^{\neq N}_\nu] = m_N\eta^{\mu\nu}\mathcal{V}_\nu$ - $m_N\{(1+\sqrt{1-v^2}B^{\neq N}_o)^{-1}(B^{\neq N}_o + B^{\neq N}_i v^i); 0\} = m_N(1-v^2)^{-1/2}\{1+(g^{-1/2}_{oo} - 1 + g_i v^i); v^i\}$ and $P_{N\mu}P^\mu_N = m^2_N(\mathcal{V}_\mu\mathcal{V}^\mu + 0) = m^2_N$ for arbitrary gravitational four-potential $B^{\neq N}_{N\mu}$.

Substituting the metric tensor (38) into (36), we obtain a general equation for the proper four-interval, $ds = ds_N$, of any selected cone object N,

$$ds^2 + dl^2 = (dx^o + \sqrt{1-v^2}B^{\neq N}_\mu \dot{x}^\mu ds)^2, \tag{39}$$

where $\dot{x}^\mu \equiv dx^\mu/ds$ and $dl^2 \equiv \delta_{ij}dx^i dx^j$. Each elementary forming-up potential $a_{K\mu}(x)^{s_K=o}_{x\neq\xi_K}$ within the gravitational four-potential $B^{\neq N}_\mu(x)$ satisfies the Maxwell-type equation with the proper metric tensor, determined by all external fields for m_K. This elementary potential takes the static Newton - Coulomb components $\{r^{-1}_K; 0\}$ in a local rest frame of this object K. The radius $r_K \equiv r_K(x, \xi_K[\tau_K])^{s_K=o}_{x\neq\xi_K} \neq 0$ is associated with the "material" parameter τ_K determined by a zero four-space interval, $s_K(x, \xi_K[\tau_K]) = 0$.

Now we derive a planetary perihelion precession in order to test the four-interval equations (39) or (36) with the new structure of the metric tensor (38), which is consistent with flat three-space, $\gamma_{ij} = \delta_{ij}$. A bound system of distant external sources with every $r_K \approx r \equiv u^{-1}$ may be considered as a united source (the Sun, for example) with an effective mass M. Relativistic relations for the function $\alpha_N(ds^2, dl^2) \equiv \sqrt{1-v^2_N}B^{\neq N}_\mu \dot{x}^\mu_N$ will be derived in the next section. One may use in (39) the Newton-Coulomb potential $B_o = -GMr^{-1} \equiv \mu u \ll 1$ for non-relativistic motion, when a considered object N (a planet with a small mass $m_N \ll M$ and velocity $v^2 = dl^2/d\tau^2 \ll 1$) moves in Sun's rest frame,

where $B_i = 0$. The equation (39) for the proper four-interval ds reads

$$ds^2(dt, dl) + dl^2 = d\tau^2(dt, dl, ds) = dt^2\left(1 - \mu u\sqrt{1 - \frac{dl^2}{d\tau^2}}\right)^2$$

$$\approx dt^2\left((1-\mu u)^2 + \mu u(1-\mu u)\frac{dl^2}{d\tau^2}\right) = dt^2(1-\mu u)^2 + dl^2\mu u(1-\mu u)^{-1}, \quad (40)$$

where we used $(dx^o)^2 = dt^2$ and $dl^2 \ll d\tau^2$.

The mass dependent coefficient at the three-interval, $dl^2 = dr^2 + r^2 d\varphi^2 = u^{-4}du^2 + u^{-2}d\varphi^2$, does not mean departure from Euclidean three-space geometry in gravitational fields. This coefficient is associated with the direct involvement of space replacement dl into the proper time rate $d\tau(dt, dl, ds)$ in agreement with the metric tensor (38). Our gravitational time dilation and the proper time rate, $d\tau = (1 - GMu)dt$ for $v^2 \ll 1$, coincides with the general relativity time rate, $\sqrt{(1 - 2GMu)}dt$, in weak fields. But there are no Schwarzschild's divergencies in the four-interval (39) for strong fields, as will be proved below.

The Killing vectors and integrals of motion, $(1-\mu u)^2 dt/ds = E = const$ and $r^2 d\varphi/ds = L = const$ (with $\vartheta = \pi/2 = const$), are well known under the four-interval (40) with stationary coefficients in strong fields, for example [24]. By taking into account these conservation laws in (40) one obtains an equation for a rosette motion of planets under the above restrictions on their velocities,

$$(1 - 2\mu u)L^{-2} + (1 - 3\mu u)(u'^2 + u^2) = E^2 L^{-2}, \quad (41)$$

where $u' \equiv du/d\varphi$ and $\mu u \ll 1$. now one may differentiate (41) with respect to the polar angle φ,

$$u'' + u - \frac{\mu}{L^2} = \frac{9}{2}\mu u^2 + 3\mu u'' u + \frac{3}{2}\mu u'^2, \quad (42)$$

by keeping only the oldest nonlinear terms. This equation may be solved in two steps, when a linear solution, $u_o = \mu L^{-2}(1 + \epsilon cos\varphi)$, is substituted into the nonlinear terms on the right hand side of (42).

The most important correction (which is summed over century rotations of the planets) is related to the "resonance" (proportional to $\epsilon cos\varphi$) nonlinear terms. We therefore ignore in (42) all corrections, apart from $u^2 \sim 2\mu^2 L^{-4}\epsilon cos\varphi$ and $u''u \sim -\mu^2 L^{-4}\epsilon cos\varphi$. Then the approximate equation for the rosette motion, $u'' + u - \mu L^{-2} \approx 6\mu^3 L^{-4}\epsilon cos\varphi$, leads to the accepted perihelion precession, $\Delta\varphi = 6\pi\mu^2 L^{-2} \equiv 6\pi\mu/a(1-\epsilon^2)$, which was originally derived from the Schwarzschild metric for the curved three-space, for example [23-26].

It is important to emphasize that the measured result, $\Delta\varphi$, for the planet perihelion precession in weak Sun fields was derived from the nonlinear four-interval (40) under the flat three-space, rather than from the linear four-interval under the curved three-space. The four-interval equation (39) may be used only for a rest-mass object, while a similar equation for photons, $dl^2/d\tau^2 = n^{-2}$, is associated with their slowness n^{-1} in gravitational fields. By taking into account that $n^{-1} = \sqrt{g_{oo}}$ in static fields, one can explain the measured gravitational light bending by keeping flat 3D space (Appendix 2).

Thus, Euclidean 3D sub-space provides the alternative way to explain main gravitational phenomena, to construct self-contained relativity, and to overcome the known conceptual difficulties [27], associated with Schwarzschild's solutions.

Covariant form of basic equations can hold universal flat three-space, which remains common for all material objects, contrary to their proper four-dimensional manifolds. One may note that flat three-space is able to remove the conventional objection (three-space curvature) against the hypothesis [28] of electromagnetic origin of gravity. There are many other arguments why flat three-space is exclusively attractive for advanced theories of space-time [29].

Below we shall study the proper canonical pseudo-Riemannian four-space and predict a new phenomenon (electromagnetic time dilation and compression), which have never been proposed from the other couplings of gravity and electrodynamics, including [30-32]. This phenomenon is available for prompt laboratory tests and may be interesting for applications.

7. Four-space with electromechanical connections

7.1. Electromagnetic time compression and dilation

The observable motion of matter is three-dimensional in spite of the fact that various high dimensional manifolds can be employed for self-consistent description of any selected object. Geometries of the proper high dimensional manifolds differ from the universal (for all objects) geometry of the common 3D subspace. The proper metric tensor $g_{\mu\nu} \equiv \eta_{\alpha\beta} e^{\alpha}_{\ \mu} e^{\beta}_{\ \nu} \neq \eta_{\mu\nu}$ of pseudo-Riemannian four-space may take only nonzero components, but must always hold (in the developed constructions) Euclidean geometry for 3D subspace, due to the hidden degeneration, $g_{oi} g_{oj} g^{-1}_{oo} - g_{ij} \equiv \gamma_{ij} = \delta_{ij}$, for real matter. This scheme provides a simple opportunity for implication of electric charges into metrics of the proper pseudo-Riemannian four-spaces. Contrary to the conventional formalism of general relativity, there are only four degrees of freedom from ten components $g_{\mu\nu}$ due to the above six conditions for 3D metrics. One ought to expect that only four new relations, rather than ten Einstein-type equations $T^{\mu\nu}_{N} = 0$, may remain independent for both gravitation and electrodynamics in flat three-space.

In order to describe objects with electric charge and rest mass we return to (1) and (37), but with the symmetrical contribution of gravitomechanical and electric charges into the proper canonical four-momentum,

$$P_{N\mu}(x) \equiv \left\{ \frac{m_N}{\sqrt{1-v^2}} + \frac{m_N(\sqrt{g_{oo}}-1)}{\sqrt{1-v^2}}; -\frac{m_N v_i}{\sqrt{1-v^2}} - \frac{m_N g_i \sqrt{g_{oo}}}{\sqrt{1-v^2}} \right\}$$

$$\equiv m_N V_\mu \equiv m_N \delta^{\alpha}_{\mu} V_\alpha + \sum_{K}^{K\neq N} (-G m_N m_K + q_N q_K) a_{K\mu}(x)^{s_K = o}_{x \neq x_K} \qquad (43)$$

or $P_{N\mu}(x)^{s=o}_{x\neq\xi} = \{ m_N \delta^{\alpha}_{\mu} V_\alpha + m_N U^{\neq N}_{\mu}(x)\}^{s=o}_{x\neq\xi}$, with $\delta^{\alpha}_{\mu} V_\alpha \equiv \mathcal{V}_\mu = \{\beta^{-1}, -\beta^{-1} v_i\}$, $U^{\neq N}_{\mu}(x) = m_N^{-1} \sum_K^{K\neq N} (-G m_N m_K + q_N q_K) a_{K\mu}(x)^{s_K = o}_{x\neq x_K} = B^{\neq N}_{\mu} + m_N^{-1} q_N A^{\neq N}_{\mu}$, and $\beta = \sqrt{1 - \delta_{ij} v^i v^j} = ds/d\tau = ds/(ds^2 + dl^2)^{1/2}$.

By considering joint roots for the electric and gravitomechanical external fields, one can introduce (for a selected charged object N) a proper canonical four-dimensional space x^{μ}_{N} with the affine connections generated by both electric and gravitomechanical external charges. The proper canonical four-momentum in this pseudo-Riemannian four-space takes the "old", mechanical view, $P^{\mu}_{N} =$

$m_{_N} dx^\mu_{_N}/ds_{_N}$, $P_{_N\nu} = m_{_N} g^N_{\mu\nu} dx^\mu_{_N}/ds_{_N} = m_{_N}\delta^\alpha_\nu V_\alpha + m_{_N} B^{\neq_N}_\nu + q_{_N} A^{\neq_N}_\nu$, $ds^2_{_N} = g^N_{\mu\nu} dx^\mu_{_N} dx^\nu_{_N}$, $P_{_N\mu} P^\mu_{_N} = m^2_{_N}$. Then all external electric charges contribute into the proper canonical metric tensor $g^N_{\mu\nu} = \eta_{\alpha\beta} e^\alpha_{_N\mu} e^\beta_{_N\nu}$ and into its tetrad

$$e^\alpha_{_N\mu}(x)^{s_N=o}_{x\neq\xi_N} = \delta^\alpha_\mu + \delta^{\alpha o}\sqrt{1-v^2}\,U^{\neq_N}_\mu(x)^{s_N=o}_{x\neq\xi_N}. \tag{44}$$

One can verify from (44) that the canonical metric tensor is consistent with the same flat three-space, $g_{oi}g_{oj}g^{-1}_{oo} - g_{ij} = \delta_{ij}$, as well as the pure mechanical analog (37). The sole difference between the proper four-intervals in the electromechanical and mechanical pseudo-Riemannian four-spaces is related to the different proper times for charged and neutral objects in external fields. But the proper times are always different even among electrically neutral objects and the additional contribution of electrical charges into the proper time notion cannot change Einstein's covariant formalism for relativistic motion.

In the general case the proper time of the selected charged object N, $d\tau_{_N} \equiv d\tau = \beta^{-1} ds = \sqrt{g_{oo}}(dx^o - g_i dx^i) = (1+\beta U_o)dx^o + \beta U_i dx^i = dx^o + \beta U_\mu dx^\mu = dx^o + \beta^2 U_\mu P^\mu m^{-1} d\tau$, depends on all external gravitational and electromagnetic fields,

$$d\tau^2 = \left(\frac{1+\beta U^{\neq_N}_o}{1-\beta U^{\neq_N}_i v^i_{_N}}\right)^2 dt^2 = \frac{dt^2}{(1-\beta^2 U^{\neq_N}_\mu P^\mu m^{-1})^2}. \tag{45}$$

Again, any curved proper four-space (which is specific for every neutral or charged object) is only auxiliary notion. Evolution of matter takes place in three-space with the universal Euclidean geometry for all extended charges and masses. There is no common curved Universe or curved four-space for different 4D cone objects. Nevertheless one may consider a common space+time manifold $\{dt(\mathbf{x}); \mathbf{x}\}$ due to the universal one-dimensional interval (7), $dt^2_{_K} = \gamma^K_{oo} dx^o_{_K} dx^o_{_K}$ with $\gamma^K_{oo} = \delta_{oo} = 1$, and due to the common 3D subspace, which keeps three-interval $dl^2_{_K} = \gamma^K_{ij} dx^i_{_K} dx^j_{_K}$ with the universal tensor $\gamma^K_{ij} = \delta_{ij}$ for all charged objects.

The physical velocities of charged cone objects in the flat three-space, $dx^i/d\tau \equiv v^i_{_N} \equiv v^i \equiv v_i \equiv v_{_N i}$, are related with the proper time and, consequently, with a particular distribution of external sources of charges and masses. Note, that masses contribute into the external canonical potential $U^{\neq_N}_\mu$ with the same sign and they can provide only the Einstein-type time dilation [1] in (45). External electric charges with different signs can lead to the electromagnetic time compression, as well as to the electromagnetic time dilation. Both these phenomena are much stronger for macroscopic charged objects than the gravitational time dilation for these objects, and one may expect to test the proposed electromagnetic compression-dilation of time (45) in the laboratory.

There are experiments showing that the rate of radioactive α-decay may be accelerated by external electrical potential of the Van de Graaff generator [33]. There is also an observation [34] that tritium decays rapidly within the metal matrix that disagrees with the established theory of β-decay. The measured decelerated oscillations of an electrically charged torque pendulum in a Faraday cage [35] stimulate also to test (45) in practice.

By taking into account (45) with $\beta = ds/(ds^2 + dl^2)^{1/2}$, one may relate all three proper intervals $ds^2_{_N} \equiv g^N_{\mu\nu} dx^\mu_{_N} dx^\nu_{_N}$, $dl^2_{_N} \equiv \delta_{ij} dx^i_{_N} dx^j_{_N}$, and $dt^2_{_N} \equiv$

$\delta_{oo}dx^o_{_N}dx^o_{_N}$,

$$ds^2 + dl^2 = \left(\frac{ds^2 + dl^2 + ds\sqrt{ds^2 + dl^2}U_o}{ds^2 + dl^2 - U_i dx^i ds} \right) dt^2. \tag{46}$$

The relation (46) for the four-interval $ds \equiv ds_{_N}$ may be represented, due to the equalities $ds^2_{_N} + dl^2_{_N} \equiv (dx^o_{_N} + \alpha_{_N} ds_{_N})^2$ and $\alpha_{_N} \equiv \beta U^{\neq N}_\mu P_{_N\mu} m^{-1}_{_N} \equiv (U^{\neq N}_o + U^{\neq N}_i v^i)/(1 + \beta U^{\neq N}_o)$, in the following form,

$$ds^2 \equiv \left(\frac{\alpha_{_N} dx^o \pm \sqrt{(dx^o)^2 - dl^2(1 - \alpha^2_{_N})}}{(1 - \alpha^2_{_N})} \right)^2 \approx \frac{dt^2}{(1 - \alpha_{_N})^2} - \frac{dl^2}{(1 - \alpha_{_N})}, \tag{47}$$

when $(1 - \alpha^2_{_N})dl^2/dt^2 \ll 1$. Notice that $\alpha_{_N} = \alpha_{_N}(ds, dt, dl)$ and there is no Schwarzschild's peculiarity in the nonlinear equations (46)-(47) that provides a novel approach to the black hole problem.

The following summary of the main relations between the proper metric tensor, $g_{\mu\nu} \equiv g^{_N}_{\mu\nu}(x_{_N})$, the proper canonical four-momentum, $P_\mu \equiv P_{_N\mu}(x_{_N})$, and external fields, $U_\mu \equiv U^{\neq N}_\mu(x_{_N})$, may be useful for applications,

$$\begin{aligned}
g_{oo} = (1 + \beta U_o)^2,\ g_{oi} = (1 + \beta U_o)\beta U_i,\ g_{ij} = \beta^2 U_i U_j - \delta_{ij}, \\
g^i = -g^{oi} = \gamma^{ij} g_j = g_i = -g_{oi} g^{-1}_{oo} = -U_i(\beta^{-1} + U_o)^{-1} \\
g^{oo} = g^{-1}_{oo} - g_i g^i = (1 - \beta^2 U_i U_j \delta^{ij})(1 + \beta U_o)^{-2},\ \gamma_{ij} = \gamma^{ij} = -g^{ij} = \delta_{ij} \\
P_\mu = m g_{\mu\nu} dx^\nu/ds = m(\delta^\alpha_\mu V_\alpha + U_\mu) = m u_\mu + q A_\mu = m V_\mu \\
P_\mu = m\{\beta^{-1} + U_o\ ;\ -\beta^{-1} v_i + U_i\} = m(\delta^\alpha_\mu V_\alpha + U_\mu) = g_{\mu\nu} P^\nu \\
P^\mu = \{m(\beta^{-1} + U^o)\ ;\ P^i\} = m\{\beta^{-1} - (U_o + U_i v^i)(1 + \beta U_o)^{-1}\ ;\ \beta^{-1} v^i\} \\
P_\mu P^\mu = g_{oo}(P^o - g_i P^i)^2 - \delta_{ij} P^i P^j = P^2_o g^{-1}_{oo} - m^2 \beta^{-2} v^2 = m^2.
\end{aligned} \tag{48}$$

It is remarkable that the contravariant component $P^i_{_N} = m_{_N}\beta^{-1} v^i$ does not depend on external potentials at all, $U^i_{\neq N} \equiv 0$, contrary to the canonical three-momentum $P_{_N i} = -m_{_N}\beta^{-1} v_i + m_{_N} U^{\neq N}_i$. This means that the external electromagnetic potentials $A^{\neq N}_\mu$ and $A^\mu_{\neq N}$, as well as the external gravitoelectromagnetic potentials $U^{\neq N}_\mu, U^\mu_{\neq N}$, are not four-vectors in the proper four-space $x^\mu_{_N}$. The proper four-space $x^\mu_{_N}$ and its proper metric tensor $g^{_N}_{\mu\nu}$ were introduced for the proper notions of one selected object N, but not for its external fields. In our approach the proper metric tensor cannot be applied for rising or lowering indexes of external fields, *i.e.* $A^{\neq N}_\mu \neq g^{_N}_{\mu\nu} A^\nu_{\neq N}$ and $U^{\neq N}_\mu \neq g^{_N}_{\mu\nu} U^\nu_{\neq N}$ (as well as $P_{_K\mu} \neq g^{_N}_{\mu\nu} P^\nu_{_K}$). As a result, a pure electrical object with $q_{_N} \neq 0$ and $m_{_N\mu} = 0$ can not exist in reality.

Note that the conventional scalar product of any canonical four-vectors in classical electrodynamics, for example $g^m_{\mu\nu}(mu^\mu + qA^\mu)(mu^\nu + qA^\nu) = m^2 + 2mq g^m_{\mu\nu} u^\mu A^\nu + q^2 g^m_{\mu\nu} A^\mu A^\nu$, is not associated with conservations (because the pure mechanical metric relations, $g^m_{\mu\nu} u^\mu u^\nu$ =1, are incorrect for electrically charged objects). This fact prevented to a reasonable introduction of the canonical four-momentum as an independent variable in the classical theory, which operates with the point particle in the collective field, rather than with external fields for the proper field densities of every extended object.

Again, the proper metric tensors $g^{\mu\nu}_{_N}$ can not be applied for rising indexes of any one summand in $m_{_N} V_\mu = m_{_N} u_\mu + q_{_N} A^{\neq N}_\mu = m_{_N} \mathcal{V}_\mu + m_{_N} B^{\neq N}_\mu + q_{_N} A^{\neq N}_\mu$,

despite $g^{\mu\nu}_{_N} V_\mu = V^\nu$ for the proper four-velocity $V_\mu \equiv V_{_N\mu}$ of the charged object N. Proper four-space and its geometry or metric tensor is associated only with proper notions of a considered material object, rather with its external fields $U^{\neq_N}_\mu = \sum_{_K}^{K\neq_N} C_{_K} a_{_K\mu}$. There are proper tensors $g^{_K}_{\mu\nu}$, which relate the four-vectors $a_{_K\mu}$ and $a^\mu_{_K}$ in every four-space $x^\mu_{_K}$, but their is no common or universal metrics, which relates the quantities without definite tensor nature, $U^{\neq_N}_\mu$ and $U^\mu_{\neq_N}$.

The four-vector $P_{_N\mu} = m_{_N} g^{_N}_{\mu\nu} dx^\nu_{_N}/ds_{_N} = m_{_N} u_\mu + q_{_N} A^{\neq_N}_\mu$ takes the unified structure, which cannot be informally split in the proper four-space $x^\mu_{_N}$ into independent mechanical and electromagnetic four-vectors. Neither of the two summands, $m_{_N} u_\mu$ and $q_{_N} A^{\neq_N}_\mu$ is a four-vector in the canonical four-space. One may verify from (48) that $m_{_N} u_\mu m_{_N} u^\mu \neq m^2_{_N}$ for any electrically charged particle in external electromagnetic fields. It is important to underline, that the action (3b), for example, is associated with the scalar product of the two four-vectors in the proper canonical four-space $x^\mu_{_N}$, because $P_{_N\mu} i^\mu_{_N} \to (m_{_N} g^{_N}_{\mu\nu} dx^\nu_{_N}/ds_{_N}) dx^\mu_{_N}$ $= m_{_N} ds_{_N}$.

Contrary to different geometries of curved four-spaces, Euclidean geometry for flat 3D sub-spaces may be universally introduced for all objects. This opens a way for comparisons, measurements and observations of different matter evolution in common 3D space. In actual practice, one may measure only a flat 3D interval, $dl = \sqrt{\gamma^{_N}_{ij} dx^i dx^j}$, and a flat 1D interval, $dt = \sqrt{\gamma^{_N}_{oo} dx^o dx^o}$, due to the universal relations $\gamma^{_N}_{ij} = \delta_{ij}$ and $\gamma^{_N}_{oo} = \delta_{oo}$ for all objects. One cannot measure a 4D interval $ds_{_N} = \sqrt{g^{_N}_{\mu\nu} dx^\mu dx^\nu}$, as well as u_μ or V_μ, because $g^{_N}_{\mu\nu}$ for measured object N does not coincides, $g^{_N}_{\mu\nu} \neq g^{_K}_{\mu\nu}$, with metrics for other objects or instruments. In other words a curved four-space space $x^\mu_{_N}$ is an auxiliary notion, and one may measure only dl/dt, rather than the mechanical four-velocity u_μ or canonical four-velocity V_μ.

Now a natural question arises: Does equivalence principle take place in the proper four-space with unified electromechanical connections?

7.2. Equivalence principle for charges

Virtual and material variations δx^μ in the action (3a) lead to geodesic conservations of the canonical four-momentum $P_{_N\mu}$ at all material points of the selected object N,

$$DP_{_N\mu}(x)^{s[\tau]=o}_{x\neq\xi[\tau]} = 0. \tag{49}$$

These geodesic equations correspond to the equivalence principle for the charged object N in its proper four-space with electromechanical connections. One may rewrite (49) in an equivalent form,

$$\{P^\nu_{_N}(x) W_{_N\nu\mu}(x)\}^{s[\tau]=o}_{x\neq\xi[\tau]} = 0, \tag{50}$$

because $P^\nu_{_N} \nabla_\mu P_{_N\nu} = 0$ and $DP_{_N\mu} \equiv dx^\nu_{_N} \nabla_\nu P_{_N\mu} = ds_{_N} m^{-1}_{_N} P^\nu_{_N} \nabla_\nu P_{_N\mu} = ds_{_N} m^{-1}_{_N} P^\nu_{_N} W^{_N}_{\nu\mu} = 0$ at all material points, $s[\tau] = 0$ and $x \neq \xi_{_N}$. Now one may use the formal separation of the canonical tensor (14) into the three non-tensor summands in these geodesic equations, $P^j(\partial_j P_o - \partial_o P_j) = 0$ and $P^j(\partial_j P_i - \partial_i P_j) + P^o(\partial_o P_i - \partial_i P_o) = 0$, in order to separate the gravitational and electromagnetic forces under free motion of charges,

$$P^\mu_{_N}(\partial_\nu \mathcal{V}_\mu - \partial_\mu \mathcal{V}_\nu) = P^\mu_{_N}(\partial_\mu B^{\neq_N}_\nu - \partial_\nu B^{\neq_N}_\mu) + q_{_N} m^{-1}_{_N} P^\mu_{_N}(\partial_\mu A^{\neq_N}_\nu - \partial_\nu A^{\neq_N}_\mu), \tag{50$'$}$$

with $\mathcal{V}_\mu = \delta^\alpha_\mu V_\alpha = \{\beta^{-1}, -\beta^{-1}v_i\}$. This is a replacement of the Minkowski-Lorentz equation, $-m_{_N} Du^\mu/ds \equiv m_{_N} u^\mu(\nabla_\nu u_\mu - \nabla_\mu u_\nu) = q_{_N} u^\mu F^{\neq N}_{\mu\nu}$, for electric charges in the conventional four-space with pure gravitomechanical connections, when $u_\mu u^\mu = 1$. The Lorentz force, $q_{_N} m^{-1}_{_N} P^\mu_{_N} F^{\neq N}_{\mu\nu}(x)$, is accompanied by its gravitational analog for masses, $P^\mu_{_N} G^{\neq N}_{\mu\nu}(x)$, in (50'), where $q(E+v\times B)_i + m(E_g + v\times B_g)_i = m\beta^{-1}[\partial_i\mathcal{V}_o - \partial_o\mathcal{V}_i - \mathcal{V}^j(\partial_j\mathcal{V}_i - \partial_i\mathcal{V}_j)] \approx m[\partial_o v_i + (v^j\nabla_j)v_i] = mdv_i/dt$ and $m_{_N} v^i dv_i/dt = v^i(m_{_N} E_{gi} + q_{_N} E_i)$ for the nonrelativistic limit. There are no formal discrepancies with Newton's and Lorentz's forces in this limit, but the self-action forces, $P^\mu_{_N} m_{_N} f_{_N\mu\nu}(x)$ and $q^2_{_N} m^{-1}_{_N} P^\mu_{_N} f_{_N\mu\nu}(x)$, are absent under the geodesic motion (49)-(50'). The absence of self-action or self-acceleration of free charges in practice corresponds to restrictions $P^\mu_{_N} f_{_N\mu\nu} = 0$ or $dx^\mu_{_N} f_{_N\mu\nu} = 0$, which can be employed in the present scheme along with the geodesic relations (50).

7.3. Maxwell-type equation for vector electrogravity

The energy-tensor $T^{\mu\nu}_{_N}(x)^{s[\tau]=0}_{x\neq\xi[\tau]}$ for the material cone-object is defined by (29) with $dx^\mu_{_N}/ds_{_N} = P^\mu_{_N}$,

$$T^{\mu\nu}_{_N} \equiv \{\frac{P^\mu_{_N} I^\nu_{_N} + P^\nu_{_N} I^\mu_{_N}}{2} + \frac{W_{_N\rho\lambda}}{16\pi}[g^{\mu\nu}_{_N} f^{\rho\lambda}_{_N} - 2g^{\mu\rho}_{_N} f^{\nu\lambda}_{_N} - 2g^{\nu\rho}_{_N} f^{\mu\lambda}_{_N}]\}^{s[\tau]=0}_{x\neq\xi[\tau]}. \qquad (29')$$

Should this energy-tensor take only zero components as it may be accepted for the conventional case, based on the supposed, not proven, independency of ten components in $g^{_N}_{\mu\nu}$? Now one may exam (from (48), for example) the relations between components $P_{_N\mu}$ and $g^{_N}_{\mu\nu}$, with $P_{_N\mu}P_{_N\nu}g^{\mu\nu}_{_N} = m^2_{_N}$. In the general case both $P_{_N\mu}$ and $g^{_N}_{\mu\nu}$, or the proper tetrad (44), depend on the same external four-field $U^{\neq N}_\mu$, while $I^\mu_{_N}\delta P_{_N\mu} = m_{_N} I^\mu_{_N}\delta[g_{\mu\nu}dx^\nu(g_{\rho\lambda}dx^\rho dx^\lambda)^{-1/2}] = (P^\mu_{_N} I^\nu_{_N} + P^\nu_{_N} I^\mu_{_N})\delta g_{\mu\nu}/2$ for arbitrary proper four-vector $I^\mu_{_N}$. First, the metric tensor cannot be irrelevant under variations of the action (3a) with respect to $P_{_N\mu}$ that was mentioned under derivation of the Maxwell-type equations (5). Second, not all components of $g^{_N}_{\mu\nu}$ are independent from one another, and the Hilbert variation for the Einstein-type equation, $T^{\mu\nu}_{_N} = 0$, is not a well defined procedure.

The first general set (15) of Euler-Lagrange equations was derived after the variations of the action over changes of the proper field $a_{_N\mu}$. Variations resulting from changes of the external field $U^{\neq N}_\mu$, or any of its summand $a_{_K\mu}$, may lead, in principle, to the other set of four Euler-Lagrange equations. But it is important to remind that external fields are not proper four-vectors in the curved four-space $x^\mu_{_N}$ and they cannot be employed as variables for the scalar action (3a).

An appropriate proper variable is the four-vector $P_{_N\mu}$, with $\delta P_{_N\mu} = \delta U^{\neq N}_\mu$ in the frame of reference with $v_i = 0$ and $\beta = 1$. Note that the proper tetrad ${e^o}_\mu = e_{o\mu}$, (44), is proportional to the proper four-momentum (43) in this frame, that may be used under variation of (3b),

$$\delta S = -\int d^4x\sqrt{-g}T^{\mu\nu}_{_N}\delta(e^\alpha_{_N\mu}e^{_N}_{\alpha\nu}) = -\int d^4x\sqrt{-g}2T^{\mu\nu}_{_N}e^{_N}_{\alpha\nu}\delta(\delta^\alpha_\mu + \delta^{\alpha o}U^{\neq N}_\mu)$$

$$= -\int d^4x\sqrt{-g}2T^{\mu\nu}_{_N}e^{_N}_{o\nu}\delta U^{\neq N}_\mu = -\int d^4x\sqrt{-g}2m^{-2}_{_N}T^{\mu\nu}_{_N}P_{_N\nu}\delta P_{_N\mu} = 0. \quad (51)$$

The action ought to be constant with respect to changes of external fields and corresponding variations of proper variables. Twelve, from sixteen, variational components in (51) are absent, because $\delta(e^b{}_\mu) \equiv \delta(\delta^b_\mu) = 0$ when $b = 1,2,3$. The variations in (51) lead to a vector Euler-Lagrange equation,

$$T^{\mu\nu}_{N}(x)P_{N\mu}(x)^{s=o}_{x\neq\xi} = 0, \tag{52}$$

rather than to a tensor Einstein-type equation $T^{\mu\nu}_{N}(x)^{s=o}_{x\neq\xi} = 0$. Due to its tensor nature the equation (52) is valid under transformations to frames of references with $v_i \neq 0$. By substituting (29') in (52), one finds a four-vector equation,

$$\left[I^\nu_{N} + P^\nu_{N}\frac{P_{N\mu}I^\mu_{N}}{m^2_{N}} - \frac{W_{N\rho\lambda}}{8\pi m^2_{N}}\left(2P^\rho_{N}f^{\nu\lambda}_{N} + 2g^{\nu\rho}_{N}P_{N\mu}f^{\mu\lambda}_{N} - P^\nu_{N}f^{\rho\lambda}_{N}\right)\right]^{s=o}_{x\neq\xi} = 0, \tag{53}$$

and its scalar contraction,

$$P_{N\nu}I^\nu_{N}(x)^{s=o}_{x\neq\xi} = \frac{W_{N\rho\lambda}(x)}{16\pi m^2_{N}}[4P^\rho_{N}P_{N\nu}f^{\nu\lambda}_{N}(x) - m^2_{N}f^{\rho\lambda}_{N}(x)]^{s=o}_{x\neq\xi}. \tag{54}$$

Now one may use the geodesic relations, $P^\mu_{N}W_{N\mu\nu} = 0$, and the self-action restrictions, $P^\mu_{N}f_{N\mu\nu} = 0$, in order to rewrite (54),

$$P_{N\nu}I^\nu_{N}(x)^{s=o}_{x\neq\xi} = -\frac{1}{16\pi}W_{N\rho\lambda}(x)f^{\rho\lambda}_{N}(x)^{s=o}_{x\neq\xi}, \tag{55}$$

and to derive a vector Maxwell-type equation,

$$\left\{i^\nu_{N}(x) - \frac{1}{4\pi}\nabla_\mu f^{\mu\nu}_{N}(x) + \frac{W_{N\rho\lambda}(x)f^{\rho\lambda}_{N}(x)}{16\pi m^2_{N}}P^\nu_{N}(x)\right\}^{s[\tau]=o}_{x\neq\xi[\tau]} = 0 \tag{56}$$

or $\{I^\nu_{N} - m^{-2}_{N}P_{N\mu}I^\mu_{N}P^\nu_{N}\}^{s=o}_{x\neq\xi} = 0$, which generalizes the equation (5) for the potential state N with $W_{N\rho\lambda}(x)^{s=o}_{x\neq\xi} = 0$.

Conservation of the mass or electrical charge four-current, $q_{N}\nabla_\mu i^\mu_{N} = 0$, is verified by practice, while one finds $\nabla_\nu i^\nu_{N} \equiv \nabla_\nu I^\nu_{N} = m^{-2}_{N}\nabla_\nu(P_{N\mu}I^\mu_{N}P^\nu_{N})$ from (56). In order to prove that (56) is consistent with the charge conservation, we use infinitesimal homogeneous coordinate shifts, $x'^\mu = x^\mu + \xi^\mu$, in the action (3a). This procedure, for example [8], leads to the following vector conservations for the elementary cone object,

$$\nabla_\mu T^\mu_{N\nu} = \left\{\frac{\nabla_\mu\left(P^\mu_{N}I_{N\nu} + P_{N\nu}I^\mu_{N}\right)}{2} - \frac{W_{N\nu\lambda}\nabla_\rho f^{\rho\lambda}_{N} + f_{N\nu\lambda}\nabla_\rho W^{\rho\lambda}_{N}}{8\pi}\right\}^{s=o}_{x\neq\xi} = 0, \tag{57}$$

where we used $W^{\rho\lambda}_{N}\nabla_\nu f_{N\rho\lambda} = 2W^{\mu\lambda}_{N}\nabla_\mu f_{N\nu\lambda}$, $f^{\rho\lambda}_{N}\nabla_\nu W_{N\rho\lambda} = 2f^{\mu\lambda}_{N}\nabla_\mu W_{N\nu\lambda}$ due to (12) and (16). Contraction of (57) with P^ν_{N}, under (50) and (15), reveals one more scalar condition under the geodesic motion of free cone-charges, $m^2_{N}\nabla_\mu I^\mu_{N}(x)^{s=o}_{x\neq\xi} = -P^\nu_{N}\nabla_\mu(P^\mu_{N}I_{N\nu})^{s=o}_{x\neq\xi} \equiv -\nabla_\mu(P^\mu_{N}P^\nu_{N}I_{N\nu})^{s=o}_{x\neq\xi}$. This condition is compatible with the covariant divergence of the vector (56), $\nabla_\mu I^\mu_{N}(x)^{s=o}_{x\neq\xi} = m^{-2}_{N}\nabla_\mu(P^\mu_{N}P^\nu_{N}I_{N\nu})^{s=o}_{x\neq\xi}$, only under conservation of the cone-particle four-flow, when $\nabla_\mu I^\mu_{N}(x)^{s=o}_{x\neq\xi} \equiv \nabla_\mu i^\mu_{N}(x)^{s=o}_{x\neq\xi} = 0$ and $\nabla_\mu(P^\mu_{N}P^\nu_{N}I_{N\nu})^{s=o}_{x\neq\xi} = 0$.

Potential or superfluid states without radiation or energy exchange with external matter obey the particular restrictions, $I^\mu_{N} = W_{N\mu\nu} = T^{\mu\nu}_{N} = 0$, which

satisfy the general equations (52)-(56). Both gravitation and electrodynamics of superfluid charges and their potential states, based on (5), (12), (15), (16) and (30), may be represented in a gauge invariant form with respect to the proper four-vectors $a_{N\mu}(x) \to a_{N\mu}(x) + \partial_\mu \chi_N(x)$ and $P_{N\mu}(x) \to P_{N\mu}(x) + \partial_\mu \Upsilon_N(x)$, with $\partial_\mu\partial_\nu\chi_N(x) = \partial_\nu\partial_\mu\chi_N(x)$ and $\partial_\mu\partial_\nu\Upsilon_N(x) = \partial_\nu\partial_\mu\Upsilon_N(x)$. Note that gauge transformations may be introduced only for proper four-vectors, rather than for external fields $B_\mu^{\neq N}$ and $A_\mu^{\neq N}$, which are not four-vectors in the considered four-space x_N^μ. Could one employ for a moment common metrics for all four-spaces x_K^μ (pseudo-Euclidean common four-space in classical electrodynamics, for example), gauge transformations might be considered for external potentials, as is known.

Radiation or energy exchange with external matter is not allowed for the potential motion. Non-potential motion ought to satisfy the general equations (56), (12), (15), (16), and (50). The energy-tensor density may have nonzero components, but $T_N^{\mu\nu} P_{N\mu} = 0$ under general motion in vector electrogravity. Emission and absorption of real photons, rather than gauge bosons, violate the gauge invariance, associated in the present scheme only with the proper fields under superfluid, potential states.

Different solutions of the general variational equations (15), (49) and (56) may be used to classify various forms of matter motion. Both gravitational and electromagnetic waves are associated with vector photons in the developed unification. These vector waves may modulate the four-space metric tensor $g_{\mu\nu}$, but they are irrelevant to Euclidean metrics, $\gamma_{ij} = \delta_{ij}$, of three-space. Vector electrogravity is consistent with the absence of space modulations in all known experiments [26] and predicts flat laboratory metrics for gravitational waves and all other relativistic phenomena.

According to conventional theory the electromagnetic and gravitational parts of interactions cannot compensate each other in the state of general motion due to the different tensor nature of these interactions. But interactions with external gravitational and electric charges are determined by the same term $U_\mu^{\neq N} = m_N^{-1} \sum_K^{K\neq N} (q_N q_K - G m_N m_K) a_{K\mu}$ in the developed vector unification. The masses m_K with their vector fields $a_{K\mu}$, rather than the energy tensor density, are sources of gravity. The universal compensation of external fields $B_\mu^{\neq N}$ and $A_\mu^{\neq N}$ for a two-body system with $q_1 q_2 = G m_1 m_2$, for example, is drastically different from the case of general relativity. The novel, vector field nature of gravity in VEG, is consistent with the attractive opportunity [28] of electromagnetic origin of the extended mass m_K, that may be studied under further developments of the present theory.

8. Conclusion

Our non-dualistic approach to the extended object was initiated by the introduction of the elementary cone-particle and the elementary cone-field in terms of a multifractional field emanating from one point source. Every object N with a rest-mass m_N contains its proper forming-up field $a_{K\mu}(x)$ at all points of the proper four-space $x^\mu = x_N^\mu$, which are related by zero four-intervals with respect to each other and the joint point vertex ξ. Proper fields $a_{K\mu}(x)^{s_N = o}_{x\neq\xi_N}$ of other extended objects K are external for the selected object N when they cross its family of charged material points, called as a light cone.

The proper four-space, $dx^{\mu}_{N} = g^{\mu\nu}_{N} dx^{N}_{\nu}$, with electromechanical connections ought to be personally introduced for every charged object. Pseudo-Riemannian metric tensor of this proper four-space, $g^{N}_{\mu\nu}(x) = \eta_{\alpha\beta} e^{\alpha}_{N\mu} e^{\beta}_{N\nu}$, with $e^{\alpha}_{N\mu} = \delta^{\alpha}_{\mu} + \delta^{\alpha o}\sqrt{1-v^2_N}\sum^{K\neq N}_{K}(-Gm_K + m^{-1}_{N} q_N q_K) a_{K\mu}(x)^{s_K=o}_{x\neq\xi_K}$, $g_{oi}g_{oj}g^{-1}_{oo} - g_{ij} \equiv \gamma_{ij} = \delta_{ij}$, $\sqrt{-g} = \sqrt{g_{oo}}$, is determined by local densities of charged fields generated by distant external sources. These densities of external fields contribute to the proper canonical four-momentum density of the selected object, $P_{N\mu}(x)^{s=o}_{x\neq\xi} \equiv m_N g^{N}_{\mu\nu} dx^{\mu}/ds = m_N \delta^{\alpha}_{\mu} V_{\alpha} + m_N \sum^{K\neq N}_{K}(-Gm_K + m^{-1}_{N} q_N q_K) a_{K\mu}(x)^{s_K=o}_{x\neq\xi_K}$. The action of the cone-object depends on the vorticity tensors $W_{N\mu\nu}(x) = \partial_{\mu}P_{N\nu}(x) - \partial_{\nu}P_{N\mu}(x)$ and $f_{N\mu\nu}(x) = \partial_{\mu}a_{N\nu}(x) - \partial_{\nu}a_{N\mu}(x)$ at all proper material points $x = x_N$, with $x \neq \xi[\tau]$ and $s(x,\xi[\tau]) = 0$.

Mirror cone particles K_1 and K_2 may be described by introducing the opposite parametric differentials $dt_{1,2}$ at every point of three-space $\mathbf{x}$. Both mirror cones for matter and for antimatter with one joint vertex (excluded from material cone states) in four-space $\{x^o; x^i\}$ contain their own field and particle fractions. The elementary cone-field and the elementary cone-particle are locally bound at every material point of their joint geometrical structure. One can apply two mirror space+time manifolds, $\{dt_1; \mathbf{x}\}$ and $\{dt_2; \mathbf{x}\}$ with $dt_1 = -dt_2$, for symmetrical evolution of matter and antimatter (in agreement with CPT symmetry) under only retarded their emission from all point sources.

Two tensor equalities (12), (16) and five vector equations $\nabla_{\mu} W^{\mu\nu}_{N}(x)^{s_N=o}_{x\neq\xi_N} = P_{N\mu} T^{\mu\nu}_{N}(x)^{s_N=o}_{x\neq\xi_N} = P^{\mu}_{N} W_{N\mu\nu}(x)^{s_N=o}_{x\neq\xi_N} = P^{\mu}_{N} f_{N\mu\nu}(x)^{s_N=o}_{x\neq\xi_N} = \nabla_{\mu} T^{\mu}_{N\nu}(x)^{s_N=o}_{x\neq\xi_N}$ $=0$ may propose the novel description of charged objects in vector gravitational and electromagnetic fields. The total scheme of vector electrogravity is self-consistent and demonstrates the unified foundation for elementary gravitational and electromagnetic proper fields - they are associated with the constant field densities, m_N and q_N, on the basis of the same forming-up cone-field $a_{N\mu}(x)^{s_N=o}_{x\neq\xi_N}$. Both kinds of extended cone-charges cannot curve Euclidean three-space, but they are responsible for the curved four-space and for the proper time dilation-compression. The electric charge cannot exist without the mass and electromagnetic part of the canonical four-momentum cannot be considered independently as a proper four-vector.

Contrary to curved pseudo-Riemannian four-spaces x^{μ}_{N} with different proper metric tensors $g^{N}_{\mu\nu}$, all their proper 3D subspaces ${x^i}_N$ exhibit universal Euclidean geometry, $dl^2_K = \gamma^{K}_{ij} dx^{i}_{K} dx^{j}_{K}$ and $\gamma^{K}_{ij} = \delta_{ij}$. The proper time interval is also independent from proper characteristics of different objects, $dt^2_K = \gamma^{K}_{oo} dx^{o}_{K} dx^{o}_{K}$ and $\gamma^{K}_{oo} = \delta_{oo} = 1$. For this reason the common space+time manifold $\{dt, x^i\}$ (not a joint curved four-space $\{x^o; x^i\}$ with the common metric tensor for all particles, like in general relativity) may be introduced for the description of evolution of all objects in the flat three-dimensional Universe.

It is important to underline that vector electrogravity essentially differs from general relativity in the description of gravity. All interactions are linear in VEG and superposition principle is satisfied with respect to the sources. The origin of gravity is the extended mass in the present scheme, but not the stress-energy tensor. The four-component vector equation $T^{\mu\nu}_{N} P_{N\nu} = 0$, rather than the ten "independent" components of the tensor equation $T^{\mu\nu}_{N} = 0$, is responsible for both gravitation and electrodynamics in the most general case.

Being unified with electrodynamics, vector gravitation of extended masses

becomes a self-contained theory, which may be applied to practice without references of other gravitomechanical theories. Nevertheless, the developed electrodynamic approach to gravitation coincides completely with Newton's theory in the nonrelativistic limit. The available observations for all known kinds of interactions and conservations do not contradict the employed Euclidean 3D geometry for the united space-charge-mass continuum.

The developed linear synthesis of external electromagnetic and gravitational fields (under the nonlinear proper four-interval), and the integration of the cone-particle into the very cone-field structure satisfy the predicted double unified criterion [11], as well as the other known criteria [20,21] for the unified field theory. The nonmaterial point sources (*i.e.* peculiarities of matter) are excluded from the material field equations in agreement with Einstein's approach [36] to the continuum theory, and all physical magnitudes of vector electrogravity are free from divergencies.

The advanced, but not retarded, emission of charged fields from point sources and self-acceleration of extended charges are absent in electrogravity, contrary to classical electrodynamics. Both gravitation and electrodynamics of superfluid charges, based on (5), (15), and (49), take the gauge-invariant form. Electrogravity of extended masses in flat three-space is consistent with the hypothesis about the electromagnetic origin of gravitation [28], while Schwarzschild's three-space curvature around the point particle cannot be satisfactorily agreed with the electromagnetic nature of mass.

As to experiments, all gravitational observations correspond to the introduced concept of flat 3D space and flat 1D time intervals for 4D cone objects. The extended electric cone-charge (and cone-mass) is consistent with the celebrated Aharonov-Bohm phenomenon [15] and the relativistic experiments with rotating superconductors [14]. A practical opportunity to test the proposed scheme for electrogravity in the laboratory is to verify the predicted electromagnetic compression-dilation of time for charged objects, like in the experiments [33-35].

The introduced concept of the extended cone-charge rejects the classical three-space with bulk, particle free regions. Material intersections of curved charged four-spaces on flat 3D subspaces may clarify the "action-at-a-distance" and energy transfer within the common space-charge-mass continuum. Vector nature of electrogravity corresponds to the unified, photonic way for all kinds of electromagnetic and gravitational radiations, and predicts the absence of metric modulations of flat, laboratory space under the forthcoming search of gravitational waves [37].

Appendix 1: applications of the $\hat{\delta}$-operator

The $\hat{\delta}^4_{_N}(x,\xi)^{s=o}_{x\neq\xi}$-operator formalism for the conjugation of the point source at ξ with the infinite material continuum at $s(x,\xi) = 0$ and $x \neq \xi$ may be demonstrated for flat four-space ($g^{\nu\nu} = 1, g^{ij} = \delta^{ij}$, $g_{ii} = -1, g_{oo} = 1$, $d\xi_\mu = \eta_{\mu\nu} d\xi^\nu$ and $a_{_N\mu}(x,\xi) = \eta_{\mu\nu} a^\nu_{_N}(x,\xi)$, $\eta_{_N}(x,\xi)_{x\neq\xi} = \eta_{_N}(|x-\xi|)_{x\neq\xi}$, $|x-\xi| = [(x^o-\xi^o)^2 - (\mathbf{x}-\boldsymbol{\xi})^2]^{1/2}$), where the equation (5) can be simplified, $\partial_\mu\partial^\mu a^\nu_{_N}(x,\xi)^{s=0}_{x\neq\xi} = 4\pi i^\nu_{_N}(x,\xi)^{s=0}_{x\neq\xi}$, due to the subsidiary condition $\partial^\mu\partial^\nu a_{_N\mu} \equiv \partial^\nu\partial^\mu a_{_N\mu} = \partial^\nu \int d\xi_\mu\ \partial^\mu \eta_{_N}(|x-\xi[p]|) = -\partial^\nu\{\eta_{_N}(|x-\xi[p]|)|^{p=+\infty}_{p=-\infty}\} = 0$.

The simplified equation (5) may be solved via Green's function $G(x,x')_{x\neq x'} = \{\delta(x^o - x'^o \mp |\mathbf{x}-\mathbf{x}'|)/|\mathbf{x}-\mathbf{x}'|\}_{x\neq x'}$, associated with equation $\partial_\mu \partial^\mu G(x,x')_{x\neq x'} = \hat{\delta}^4(x,x')_{x\neq x'} \equiv \{\hat{\delta}^3(\mathbf{x},\mathbf{x}')\delta(x^o - x'^o \mp |\mathbf{x}-\mathbf{x}'|)\}_{x\neq x'}$. The direct and the inverted elementary fields are associated with different contravariant four-vectors,

$$a^\nu_{_N}(x)^{s[\tau_{1,2}]=0}_{x\neq\xi[\tau_{1,2}]} = \int d^4x' i^\nu_{_N}(x')^{s'[\tau_{1,2}]=0}_{x'\neq\xi[\tau_{1,2}]} G(x,x')_{x\neq x'}$$

$$= \int dp \int d^4x' \frac{dx'^\nu}{dp} \hat{\delta}^4_{_N}(x',\xi[p])_{x'\neq\xi} G(x,x')_{x\neq x'} = \int dp \frac{d\xi^\nu[p]}{dp} G(x,\xi[p])_{x\neq\xi}$$

$$= \int \frac{dp}{|\mathbf{x}-\boldsymbol{\xi}[p]|_{x\neq\xi[p]}} \frac{d\xi^\nu[p]}{dp} \frac{\delta(p-\tau_{1,2})}{\left| \frac{\partial\xi^o[p]}{\partial p} \pm \frac{\partial|\mathbf{x}-\boldsymbol{\xi}[p]|}{\partial p} \right|_{x\neq\xi[p]}} =$$

$$\frac{d\xi^\nu[\tau_{1,2}]/d\tau_{1,2}}{\left| r[\tau_{1,2}]\frac{\partial\xi^0[\tau_{1,2}]}{\partial\tau_{1,2}} \mp \mathbf{r}[\tau_{1,2}]\frac{\partial\boldsymbol{\xi}[\tau_{1,2}]}{\partial\tau_{1,2}} \right|^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}$$

$$= \frac{d\xi^\nu[\tau_{1,2}]}{|r[\tau_{1,2}]|\frac{d\tau_{1,2}}{|d\tau_{1,2}|}\left| d\xi_o[\tau_{1,2}]\left(1 \mp \frac{\mathbf{r}[\tau_{1,2}]}{r[\tau_{1,2}]}\frac{d\boldsymbol{\xi}[\tau_{1,2}]}{d\xi^o[\tau_{1,2}]}\right)\right|^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}}$$

$$= \frac{d\xi^\nu[\tau_{1,2}]}{|r[\tau_{1,2}]|\frac{d\tau_{1,2}}{|d\tau_{1,2}|}\,|d\xi_o[\tau_{1,2}] \mp d|\boldsymbol{\xi}[\tau_{1,2}]-\mathbf{x}||^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}} = \frac{d\xi^\nu[\tau_{1,2}]}{dt_{1,2}|r[\tau_{1,2}]|^{s[\tau_{1,2}]=0}_{\mathbf{x}\neq\boldsymbol{\xi}[\tau_{1,2}]}},$$

where we used $\mathbf{r}[\tau]d\boldsymbol{\xi}[\tau] = -r^i[\tau]d\xi_i[\tau]$, $r^i[\tau] = x^i - \xi^i[\tau]$, $|\mathbf{x}-\boldsymbol{\xi}[\tau]| \equiv r[\tau] = \sqrt{-r_i[\tau]r^i[\tau]}$, $\xi^0[\tau_1] = x^0 - r[\tau_1]$, $\xi^0[\tau_2] = x^0 + r[\tau_2]$, and $\delta[f(p)] = \delta(p-\tau)/|f'(p)|$ with $f(\tau)=0$. Two different four-potentials $a^\nu_{_N}(x)^{s[\tau_{1,2}]=0}_{x\neq\xi[\tau_{1,2}]}$ can be represented in the Lienard-Wiechert form with the velocity $d\xi^\nu[\tau_{1,2}]/dt_{1,2}$ of point sources of matter and antimatter, respectively, in the mirror space-time manifolds.

In order to derive the covariant four-potentials from their definition, one should solve (8), $d\xi^\nu[p]\partial_\mu\partial^\mu\eta_{_N}(|x-\xi[p]|)_{x\neq\xi} = 4\pi dx^\nu\hat{\delta}^4_{_N}(x,\xi[p])_{x\neq\xi}$ (in flat four-space), via the same Green's function $G(x,x')_{x\neq x'}$,

$$\eta_{_N}(|x-\xi[p]|)_{x\neq\xi[p]} = \int d^4x'\hat{\delta}^4_{_N}(x',\xi[p])_{x'\neq\xi[p]} G(x,x')_{x\neq x'}\frac{dx'^\nu}{d\xi^\nu[p]}$$

$$= G(x,\xi[p])_{x\neq\xi[p]} = \left\{\frac{\delta(x^o - \xi^o[p] \mp |\mathbf{x}-\boldsymbol{\xi}[p]|)}{|\mathbf{x}-\boldsymbol{\xi}[p]|}\right\}_{x\neq\xi[p]}.$$

By using this relation directly in the definition of the basic covariant four-potential, $a_{_N\nu}(x)^{s[\tau]=0}_{x\neq\xi[\tau]} = \int dp G(x,\xi[p])_{x\neq\xi[p]}\{d\xi_\nu[p]/dp\}$, one can verify the above derived result for both the direct (for matter) and the inverted (for antimatter) solutions with dt_1 and dt_2, respectively.

Appendix 2: Gravitational light bending

A four-interval equation for electromagnetic waves in nonstationary gravitational fields may be derived from the light velocity, $dl/d\tau = n^{-1}$ or

$$\delta_{ij}dx^i dx^j = n^{-2}g_{oo}(dx^o - g_i dx^i)^2,$$

where the slowness $n^{-1} \equiv 1/n(g_{oo}, g_i)$ is associated with the metric contribution into the Maxwell and wave equations. This slowness defines the wave and ray equations, $k_\mu k^\mu = (1-n^2)(k_\mu V^\mu)^2$ and $\delta \int k_\mu dx^\mu = 0$, for light in arbitrary gravitational fields [38].

Below we consider for simplicity only a static gravitational field. It formally works like a static medium for electromagnetic waves, because $\mathbf{D} = g_{oo}^{-1/2}\mathbf{E}$, $\mathbf{B} = g_{oo}^{-1/2}\mathbf{H}$, and $n^{-1} = (\tilde{\varepsilon}\tilde{\mu})^{-1/2} = g_{oo}^{1/2}$ in the covariant Maxwell and wave equations [8].

Equations for light rays in static gravitational fields may be found from Fermat's principle [38],

$$\delta \int k_i dx^i = -\delta \int \frac{k_o dl}{n^{-1}\sqrt{g_{oo}}} = -k_o \delta \int \frac{dl}{g_{oo}} = -k_o \delta \int \frac{\sqrt{du^2 + u^2 d\varphi^2}}{u^2(1-GMu)^2} = 0,$$

where $n^2 k_o k^o = k_i k^i, g_{oo}k^o = k_o = const$, $k_i = -nk_o g_{oo}^{-1/2} dx^i/dl$, $\sqrt{g_{oo}} = 1 - GMu$, $g_i = -g_{oi}/g_{oo} = 0$, $dl = \sqrt{\delta_{ij}x^i x^j} = \sqrt{dr^2 + r^2 d\varphi^2}$ ($r \equiv u^{-1}$, φ, and $\vartheta = \pi/2$ are the spherical coordinates). Note that both the non-homogeneous wave slowness, $n^{-1} = g_{oo}^{1/2} \neq const$, and the non-homogeneous frequency, $\hbar\omega = k_o g_{oo}^{-1/2} \neq const$ (red shift), are responsible for the twofold curvature of light rays in gravitational fields.

The variations with respect to u and φ leads to a couple of ray equations,

$$(1-GMu)^4 \left[\left(\frac{du}{d\varphi} \right)^2 + u^2 \right] = U_o^2 = const,$$

$$\frac{d^2u}{d\varphi^2} + u = 2U_o^2 \frac{GM}{(1-GMu)^5}.$$

A family of solutions, $u \equiv r^{-1} = r_o^{-1} sin\varphi + 2GMr_o^{-2}(1 + cos\varphi)$ and $r_o^{-1} = U_o$, for both these equations might be found in weak fields, if one ignores all terms nonlinear in $GMr_o^{-1} \ll 1$. A propagation of light from $r(-\infty) = \infty, \varphi(-\infty) = \pi$ to $r(+\infty) \to \infty, \varphi(+\infty) \to \varphi_\infty$ corresponds to the angular deflection $\varphi_\infty = arsin[-2GMr_o^{-1}(1 + cos\varphi_\infty)] \approx -4GMr_o^{-1}$ from the initial light direction. This result, derived under the flat three-interval dl, coincides with the measured deflection, $-1,66'' \pm 0.18''$ [26], of light rays by the Sun ($r_o = 6,96 \times 10^5 km$ and $4GMr_o^{-1} = 1,75''$).

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